An elementary handbook of logic

John Joseph Toohey
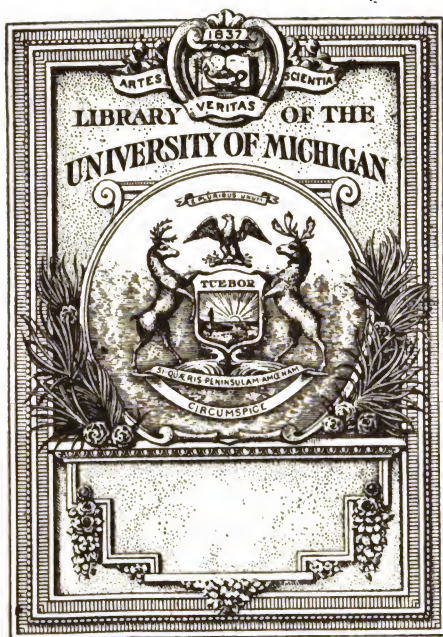
AN ELEMENTARY
HANDBOOK OF LOGIC

AN

ELEMENTARY HANDBOOK

OF

LOGIC

BY

JOHN J. TOOHEY, S. J.

PROFESSOR OF LOGIC AND METAPHYSICS IN GEORGETOWN UNIVERSITY,
WASHINGTON, D. C.

NEW YORK
SCHWARTZ, KIRWIN & FAUSS
42 BARCLAY STREET

Imprimi Potest

ANTHONY J. MAAS, S. J.
Præpositus Prov. Marylandiæ Neo-Eboracensis.

Nihil Obstat

ARTHUR J. SCANLAN, S.T.D.
Censor Librorum.

Imprimatur

✠ JOHN CARDINAL FARLEY,
Archbishop of New York.

PREFACE

The present volume makes no pretensions to being anything more than is implied in its title. It is *elementary*, and it is a *handbook*. Being elementary, it omits all the subtler questions which frequently find a place in extended treatises on Logic. Being a handbook, it is not designed for private study, but for use in the classroom. It does not attempt to provide a detailed explanation of the various topics as they come up for study. This has been left to the teacher, whose exposition of the doctrine would probably be embarrassed and rendered less effective if his pupils were confronted with long and unnecessary comments in the text.

A special effort has been made to combine clearness with brevity, so that once the student has received the teacher's explanation, he may have a lucid epitome which will enable him to recall with facility all the essential principles of Logic.

The volume departs in two particulars from the common method of treatment. First, a distinction has been drawn between the Act of Inference and the Process of Inference, and a separate chapter has been devoted to each. This will probably be found to simplify the explanation of Reasoning and to bring out the essential identity of form which underlies the various types of argument.

Secondly, the hypothesis of the Distribution of the Predicate has been abandoned. This course was

v

deemed necessary, owing to the inconsistencies which seem to beset that hypothesis. A discussion of these inconsistencies will be found in the Appendix. A second reason for this departure lay in the fact that a far more direct and simple explanation of the Categorical Syllogism is achieved by the use of the *Dicta* of the three first Figures of the Syllogism than by recourse to the hypothesis of the Distribution of the Predicate.

If it be permitted to make a suggestion to those who decide to adopt this volume as a text-book for their classes, it is that the *Dicta* be printed on charts which may be hung up in the classroom. This will be found to facilitate very materially the pupil's mastery of the subject. The *Dicta* need not be committed to memory, but the pupil should be required to deduce from them all the Rules and Moods of the Categorical Syllogism.

Georgetown University,
    January, 1918

TABLE OF CONTENTS

INTRODUCTION

CHAPTER I

APPREHENSION AND THE IDEA

CHAPTER II

THE TERM

vii

CHAPTER III

JUDGMENT

CHAPTER IV

THE CATEGORICAL PROPOSITION

CHAPTER V

EDUCTION

CHAPTER VI

THE ACT OF INFERENCE

CHAPTER VII

THE HYPOTHETICAL PROPOSITION

CHAPTER VIII

THE PROCESS OF INFERENCE AND THE SYLLOGISM

CHAPTER IX

THE MIXED HYPOTHETICAL SYLLOGISM

CHAPTER X

THE SIMPLE CATEGORICAL SYLLOGISM

CHAPTER XI

FIGURES AND MOODS OF THE CATEGORICAL SYLLOGISM

CHAPTER XII

GENERAL RULES OF THE CATEGORICAL SYLLOGISM

CHAPTER XIII

REDUCTION OF CATEGORICAL SYLLOGISMS

CHAPTER XIV

CHAPTER XV

THE PREDICABLES AND THE CATEGORIES

CHAPTER XVI

LOGICAL DIVISION

CHAPTER XVII

DEFINITION

CHAPTER XVIII

FALLACIES

APPENDIX

INTRODUCTION

THE DEFINITION OF LOGIC ˙

1. **Logic** is the science of valid reasoning.

Logic is a **science,** because it is a system of demonstrated truths which relate to a particular object.

A **speculative science** is a science of which the direct aim is the ascertainment of truth.

A **practical science** is a science of which the direct aim is the application to practice of the truths it ascertains.

Like Ethics and Æsthetics, Logic is a practical science. It aims at determining the laws of valid reasoning, not for the sake of the knowledge thus acquired, but in order to aid the mind in reasoning correctly and in detecting fallacies.

The **material object** of a science is the thing or things with which the science is occupied, as they exist independently of the science. Thus, the earth is the material object of Geography and Geology.

The **formal object** of a science is that aspect of the material object which is explicitly contemplated by the science. For example, the surface of the earth is the formal object of Geography, and the constitution of the earth is the formal object of Geology.

The **material object** of Logic is reasoning, the ˙

elements of which it is composed, and its expression in language.

The **formal object** of Logic is the validity of the reasoning.

The **validity** of reasoning is the logical dependence of one element in the reasoning upon the remaining element or elements (cf. 46, 54).

CHAPTER I

APPREHENSION AND THE IDEA

2. The mind has three cognitive acts, viz. **Apprehension, Judgment,** and **Reasoning.**

A **cognitive act** is an act of the mind by which something is known, that is, represented or asserted.

The **formal object** of a cognitive act is that which is explicitly represented or attained by that act.

Apprehension is a cognitive act which merely represents an object and does not involve in itself a mental assertion; thus, the act of the mind which represents "tree" or "gold" or "Cicero" is an apprehension.

Absolute apprehension is an apprehension which has for its formal object something absolute, that is, an object apart from its relations; e.g. the apprehension of "man," "animal," "America."

Comparative apprehension is an apprehension which has for its formal object a relation or an object *as* related to something; e.g. the apprehension of "father," "master," "similar," "thing as white as snow."

Simple apprehension is an absolute or a comparative apprehension considered apart from mental assertion.

The **material object** of apprehension is the thing or things which are apprehended, as they are in themselves, with all their attributes or aspects, independently of the mind's contemplation of them.

The **formal object** of apprehension is that aspect of

3

the material object which is explicitly represented by the apprehension. It may also be defined as that aspect of the material object under which the material object is explicitly represented by the apprehension. Or again, it is the material object under that aspect under which it is explicitly represented by the apprehension. Of course, the formal object of an apprehension need not be only one aspect; in many cases it is two or more.

Since almost every object outside the representation of the mind has hundreds of aspects, it is plain that hundreds of apprehensions (or ideas) may have one and the same material object, while their formal objects are all different.

Other words for **aspect** are **Attribute, Note, Form,** and the like. These words signify that which determines or marks a thing so that it can be known or recognized.

Apprehension may be viewed as an act of the mind or as a representation. Viewed as a representation, that is, as representing an object, apprehension is called an **Idea, Notion,** or **Concept.**

3. The **comprehension** of an idea is the sum-total of notes or attributes which the idea explicitly represents in the object. The sum-total of notes which are not explicitly represented by the idea, but which may be determined by an analysis of the formal object of the idea, may be called the **implicit comprehension** of the idea.

The **extension** of an idea is the sum-total of individuals or objects which are severally represented by the idea; that is, it is the sum-total of individuals of

which the idea can be predicated when they are taken one by one. These objects or individuals are called the **Inferiors** of the idea.

Usually comprehension and extension vary inversely; that is, the wider the comprehension, the narrower the extension, and *vice versa;* thus, the idea of "red man" has a wider comprehension, but a narrower extension, than the idea of "man." However, if the note which is added to the comprehension is already contained in the implicit comprehension of the idea, or necessarily characterizes the formal object which is represented by the idea, the addition of the note does not narrow the extension of the idea; thus, the idea of "mortal man" has the same extension as the idea of "man."

4. **Attention** is the application of the mind to something.

Prescission is an act of the mind by which it attends to one out of several aspects of an object without attending to the others. It would be still more accurate to say that prescission is an act of the mind by which it attends to an object under one of its aspects without attending to it under its other aspects. For example, the mind, contemplating Peter Jones, attends to the aspect "soldier" in him and does not attend to the aspects "American," "young," "handsome," or "tall." Here the mind is said to *prescind from* "American," "young," etc. Prescission requires that that which is prescinded shall not be really distinct from the material object. Two objects of thought are **really distinct** from each other, when they are **physically** separated or can be physically **separated;**

thus, the head of a man is really distinct from
his shoulders.   Hence, if we were to attend to
the head of Peter Jones without attending to his
shoulders, we should not be prescinding from his
shoulders.   On the other hand, a semicircle is convex
when viewed from one direction, and concave when
viewed from the opposite direction; yet it is not true
that one part of it is concave and the other part con-
vex: the *whole* semicircle is convex and the whole of
it is concave. ·Hence, when the mind attends to the
aspect "concave" without attending to the aspect "con-
vex," it prescinds from "convex."

It would also be an act of prescission if the mind
attended to two or· more aspects of an object without
attending to the others.   In that case the mind would
be said to attend to a complex aspect.

The thing outside the representation of the mind
which is characterized by the aspect or attribute is
called the **subject**; the aspect is usually called a **form**.

Abstraction is an act of the mind by which it at-
tends to the aspect or form obtained by prescission
and positively excludes the subject in which the form
resides; e.g. "tallness," "courage."

The name "abstraction" is also applied by many
authors to the act of prescission.

Reflection is an act of the mind by which it turns
to contemplate its own acts.

Psychological reflection is an act of the mind by
which it turns to contemplate its own acts so far as
they are acts or modifications of the soul.

Ontological reflection is an act of the mind by which
it turns to contemplate its own acts so far as they are

representations, that is, so far as they represent an object. It is also called ontological reflection, when the mind contemplates the formal object of a previous cognitive act for the purpose of analysis or comparison.

Analysis is the act of resolving the formal object of an idea or other cognitive act into its notes or elements.

Synthesis is the act of combining two or more notes into the formal object of one idea.

5. A **direct idea** is an idea which represents something outside the mind; e.g. the idea of a horse.

A **reflex idea** is an idea which represents something inside the mind; e.g. the idea of an abstraction.

6. A **clear idea** is an idea which distinguishes an object from other objects; e.g. the idea of "tree." The opposite of a clear idea is an **obscure** or **vague idea.**

A **distinct idea** is an idea which not only distinguishes an object from other objects, but also distinguishes two or more notes or aspects of the object; e.g. the idea of "tall pine tree." The opposite of a distinct idea is a **confused idea.**

Every idea is a clear idea as far as it goes. An idea is called obscure only by comparison with another idea which represents the same object more clearly.

A **comprehensive** or **adequate idea** is an idea which represents explicitly all there is to be known about an object. Only a being of infinite intelligence can have a comprehensive idea of anything.

7. A **concrete idea** is an idea which represents the form along with the subject; e.g. the idea of "man" or "white (horse)."

An **abstract idea** is an idea which represents the form without the subject; that is, it represents the form as standing by itself; e.g. the idea of "humanity" or "whiteness" or "rashness." The abstract idea is the result of abstraction.

8. A **singular** or **individual idea** is an idea which represents one determinate object or certain determinate objects; e.g. the idea of "Plato" or "this man" or "these horses."

A **universal idea** is an idea which represents severally many individual objects, and hence it can be predicated of each of them; e.g. the idea of "man" or "king" or "dog." The objects represented by a universal idea are called the **Inferiors** of the idea.

It is to be observed that the universal idea represents severally many objects, whether the mind adverts to those objects or not. The mind may attend to the one or more notes or attributes which the idea represents without attending to the various individuals which possess these notes or attributes. When a universal idea is used in a judgment, the mind sometimes adverts to the individuals which are represented by it, that is, to the extension of the idea, and sometimes it does not. When the mind attends separately to the objects in the extension of the universal idea, it uses the idea **distributively**. When it does not attend to the extension of the idea, it uses the idea **absolutely**. The distributive use of an idea is common in the subject of a judgment; the absolute use, in the predicate; e.g. "Every man is rational."

In the distributive use of a universal idea, the mind

sometimes adverts to all the individuals in the extension of the idea, as in the subject of the foregoing example; sometimes it adverts to an indeterminate individual or to an indeterminate number of the individuals in the extension, as in the example following: "Some men are wise." "Some men" in this proposition is by certain authors called the expression of a **particular idea.**

A **transcendental idea** is an idea which represents severally all objects whatever, and hence it can be predicated of each of them; e.g. the idea of "being" or "one."

A **collective idea** is an idea which represents a number of similar individuals *as* constituting one whole; e.g. the idea of "army" or "cavalry" or "senate." A collective idea may be either singular or universal; e.g. the idea of "this army," the idea of "army."

9. **Incompatible ideas** are ideas whose formal objects cannot co-exist in the same respect or in the same part in one individual; e.g. the ideas of "hot" and "cold," of "first" and "second."

The idea which represents a form or an object *as* having that form is called a **positive idea**; e.g. the idea of "combatant." The idea which represents the absence of a form or an object *as* lacking that form is called a **negative idea**; e.g. the idea of "non-combatant."

Privation is the absence or negation of a form which is found in a thing when the thing is in its normal condition or which a thing is fitted to possess; e.g. "blindness." Every privation is a negation, but not every negation is a privation. Lack of sight is a privation in a man, but a mere negation in a tree.

Contradictory ideas are a pair of ideas one of which

represents a form, and the other, the simple absence of that form; or they are a pair of ideas one of which represents an object as having a certain form, and the other, an object as simply lacking that form; e.g. the ideas of "combatant" and "non-combatant," of "metallic" and "non-metallic." Contradictory ideas are also called **complementary ideas,** because between them they comprise all things whatsoever.

Contrary ideas are a pair of ideas representing forms which in a given respect are at the extremes of opposition to each other; e.g. the ideas of "hot" and "cold" in respect to temperature, of "first" and "last" in respect to order.

Relative ideas are ideas representing objects so far as they are related to each other; e.g. the ideas of "father" and "son," of "Creator" and "creature." Here we have spoken of relative *ideas,* that is, of at least a *pair* of ideas. There is also a **relative** or **comparative idea,** which is the same as a comparative apprehension (cf. 2) ; such an idea represents a relation or an object *as* related to something; e.g. the idea of "brother," "equal," "thing larger than a man."

Contradictory, contrary, and relative ideas are all incompatible ideas; but there are some incompatible ideas which do not fall under any of these three heads; e.g. the ideas of "first" and "second," of "cold" and "lukewarm." It is, however, not unusual to classify such ideas under contrary ideas.

Disparate ideas are ideas representing forms which are not opposed to each other and are not necessarily related to each other; e.g. the ideas of "holy" and "learned," of "hot" and "yellow."

CHAPTER II

THE TERM

10. A **sign** is anything from which or by which something beyond itself is known; e.g. smoke is a sign of fire; breathing, of life; a footprint, of an animal.

Signification is the connection between the sign and the thing signified.

A **natural sign** is one whose signification comes from nature; e.g. smoke, a groan.

An **arbitrary sign** is one whose signification depends on convention, that is, on agreement between men; thus, the palm is a sign of victory.

Language, in general, is a natural sign; but any given word is an arbitrary sign.

11. A **word** is a vocal sound uttered by a man and having a signification from the free convention of men. "Word" is also applied to the letter or letters which are used to represent a word.

A **categorematic word** is a word which by itself has a determinate signification; e.g. "tiger," "red," "humanity."

A **syncategorematic word** is a word which has a determinate signification only when used along with another word; e.g. "every," "as," "from," "by."

Categorematic words are substantives and adjectives

and such words as may be used as substantives or ad-
jectives, and finally, pronouns in the nominative case.

In Logic all categorematic words are called **names**
or **terms.**

12. A **term** is the verbal expression of an idea.

A **simple term** is a term consisting of one word; e.g.
"animal."

A **complex term** is a term consisting of several
words; e.g. "rational animal."

Of the words composing the complex term one is
called the principal term, and the other word or words,
the incident term.

The **principal term** is the term which denotes the
subject of the form; thus, "animal" in the foregoing
example.

The **incident term** is the term which denotes the
form that is in the subject; e.g. "rational" in the same
example.

The incident term is either explicative or restrictive.

An **explicative** or **explanatory term** is a term which
denotes something that is found in the whole exten-
sion of the idea expressed by the principal term; e.g.
"mortal man."

A **restrictive term** is a term which denotes some-
thing that is found in only part of the extension of
the idea expressed by the principal term, and hence
it restricts that term; e.g. *"learned* man."

13. A term is **concrete** or **abstract** according as it
expresses a concrete or an abstract idea (cf. 7).

14. An **absolute term** is a term which expresses an

absolute idea; that is, it is a term which denotes an object without referring it, to another object; e.g. "tree," "table."

A **relative term** is a term which expresses a relative idea; that is, it is a term which denotes a relation or an object *as* related to another object; e.g. "father," "king," "thing stronger than iron." The other object is called the **correlative** (cf. 2, 9).

15. A **singular term** is a term which denotes one determinate object or certain determinate objects; e.g. "Plato," "this house," "these men."

A **collective term** is a term which denotes a number of similar objects taken together as constituting one whole; e.g. "group," "army."

A **common term** is a term which denotes a number of objects taken separately; e.g. "lion," "pillar."

16. A **univocal term** is a common term which expresses only one idea, and hence is applied severally to many objects in the same sense; e.g. "giraffe."

An **equivocal term** is a common term which expresses two or more ideas, and hence is applied to different objects in a different sense; e.g. "bow" applied to a nod of the head and to the forward part of a ship. For the logician a word employed in two senses in the same argument is equivalently two terms.

A univocal term is also called a **general term.**

A **distributed term** is a general term which refers explicitly to each of the objects in the extension of a universal idea; e.g. "all *men,*" "every *tree.*"

An **undistributed term** is a general term which refers explicitly to an indeterminate object or to each of

an indeterminate number of the objects in the exten-sion of a universal idea; e.g. "some *tree*," "some *men.*"

17. The **connotation** of a term is the sum-total of notes or attributes in an object which are conventionally sig-nified by the term.   Other names for connotation are **Comprehension, Intension,** and **Implication** (cf. 3).

The **denotation** of a term is the extension of the idea which it expresses; that is, it is the sum-total of objects to which the term can be severally applied in the same sense.   Denotation is also called **Extension** and **Application.**

SUPPOSITION OF TERMS

18. The **supposition** of a term is the use of a term in a proposition.

Material supposition is the use of a term without regard to what it denotes; e.g. *"Man* is a word of three letters."

Formal supposition is the use of a term to denote something; e.g. *"Man* is mortal."

Formal supposition is logical or real.

Logical supposition is the use of a term to denote an object as it is in the representation of the mind; e.g. *"Man* is a universal idea;" *"Man* is a species."

Real supposition is the use of a term to denote an object as it is outside the representation of the mind; e.g. *"Man* is rational."

Real supposition is singular, absolute, or general.

Singular supposition is the use of a term to denote a definite individual or certain definite individuals; e.g. "This *man* is learned;" "Those *men* are upright."

Absolute supposition is the use of a *general* term to denote a form or attribute considered in itself; e.g. "*Man* is mortal." This supposition regards only the *comprehension* of the idea expressed by the term and leaves out of account the extension. It is the kind of supposition which is usual in the predicate of a proposition when the predicate is a general term (cf. 8).

General supposition is the use of a general term to denote a form or attribute considered as existing in an indeterminate object or in a number of objects; e.g. "All *men* are mortal." This supposition regards both *comprehension* and *extension.*

General supposition is collective or distributive.

Collective supposition is the use of a *general* term for the objects severally denoted by it taken together; e.g. "All the *angles of a triangle* are equal to two right angles"="Angles A and B and C together are equal to two right angles."

Definite collective supposition is the use of a general term for *all* the objects severally denoted by it taken together; e.g. "All the *Apostles* are twelve."

Indefinite collective supposition is the use of a general term for an *indeterminate number* of the objects severally denoted by it taken together; e.g. "Some *soldiers* built the hut;" "Many *mosquitoes* (together) weigh a pound."

N. B.—Collective supposition must be carefully distinguished from the collective *term* (cf. 15, 8).

Distributive supposition is the use of a general term for the objects denoted by it taken separately; e.g. "Every *man* is mortal"="This man is mortal, and that

man is mortal, and that other man is mortal," and so on (cf. 8).

Distributive supposition is universal or particular.

Universal supposition is the use of a general term for each and every object denoted by it; e.g. "Every *man* is mortal."

There is a supposition, called **incomplete universal supposition,** which is sometimes mentioned in works on Logic. This supposition is the use of a general term, not for every object denoted by it, but for a specimen of every kind of object denoted by it; e.g. "Every *animal* was in Noah's Ark."

Particular supposition is the distributive use of a general term indeterminately for one or a number of the objects denoted by it; e.g. "Some *American* was chosen;" "Some *men* are wise."

Particular supposition is disjunctive or confused (vague).

Disjunctive supposition is the distributive use of a general term indeterminately for one or a number of the objects denoted by it in such a way that what is asserted can be verified in at least one individual object taken by itself; e.g. "Some *Apostle* was a traitor." This supposition is called *disjunctive,* because the proposition in which it occurs is resolvable into a disjunctive proposition (cf. 51). Thus, the example we have just used can be resolved as follows: "Either Peter was a traitor, or John was a traitor, or Judas was a traitor," etc. In disjunctive supposition what is asserted must be verified in at least one individual object which *in itself* is fixed and determined, though it is not determined by the term.

Confused or **vague supposition** is the distributive use of a general term indeterminately for one or a number of the objects denoted by it in such a way that what is asserted cannot be verified in any individual object taken by itself; e.g. "Some *eye* is requisite for seeing;" "Some *boat* is necessary for sailing." The proposition in which confused supposition occurs cannot be resolved into a disjunctive proposition. Thus, we cannot say, "Either the right eye is requisite for seeing, or the left eye is requisite for seeing;" but we may say, "Either the right or the left eye is requisite for seeing;" or we may say, "Some eye or other is requisite for seeing." Similarly, we may say, "Either the affirmative or the negative side is sure to win;" but we cannot resolve this into a disjunctive proposition. When confused supposition occurs, not only is the individual object not determined by the term, but it is not determined in itself. In confused supposition what is asserted can be verified determinately in one object, only in case all the other objects are lacking.

Divided supposition is the use of a term for something which is in existence at a different time from that indicated by the verb; e.g. "The blind see;" "The deaf hear,"="Those who were blind now see," etc. Such propositions are said to be false **in sensu composito** (that is, when the form of the subject and that of the predicate are considered as combined), and true **in sensu diviso** (that is, when the forms are considered as divided and as existing at different times).

CHAPTER III

JUDGMENT

19. **Judgment** is an act of the mind asserting that in the world of reality the formal objects of two ideas are one and the same thing or that they are different (that is, distinct) things (cf. 4) ; or more briefly, but less accurately, it is the mental assertion of the objective identity or diversity of two ideas. Judgment may be more loosely defined as an act of the mind asserting that the object represented by one idea possesses or lacks the attribute represented by another idea.

N. B.—The meaning of the first part of the definition of judgment is not that the mind asserts that the formal object of one idea is the formal object of another idea, but that the thing represented under one aspect by one idea is one and the same with the thing represented under another aspect by another idea (cf. 2).

In section 2 we saw that the formal object of a cognitive act is that which is explicitly represented or attained by that act; hence—

The **formal object** or **form** of a judgment is the objective identity or diversity of two ideas. More accurately, it is the identity or diversity (in the world of reality) of the formal objects of two ideas.

The **material object** or **matter** of a judgment are two ideas, or rather, the formal objects of two ideas.

18

The **world of reality** is the sum-total of actual and possible objects of thought which do not depend upon the mind's thought about them for being what they are, whether in themselves or in their relations. The world of reality embraces all objects which are outside the representation of the mind and all objects which have a foundation outside the representation of the mind.

A **reality** is anything that is independent of the mind's thought about it for being what it is.

An **unreality** is anything that is dependent upon the mind's thought about it for being what it is.

An **affirmative judgment** is a judgment which asserts the objective *identity* of two ideas; e.g. "Gold is yellow." An affirmative judgment is also called an **affirmation,** and is said to **affirm** something.

A **negative judgment** is a judgment which asserts the objective *diversity* of two ideas; e.g. "The horse is not rational." A negative judgment is also called a **negation,** and is said to **deny** something.

The **subject** of a judgment is the idea, or rather, the *object* of which something is affirmed or denied; for instance, "gold" and "horse" in the two foregoing examples.

The **predicate** of a judgment is that which is affirmed or denied of the subject; for instance, "yellow" and "rational" in the same examples.

The subject or predicate of a judgment may be either an idea representing an object absolutely or an idea representing an object as related to something else; that is, it may be either an absolute or a relative idea. For example, it may be "sentient thing" or it

may be "thing smaller than a bird." Thus, we may say, "The ant is a sentient thing," and "The ant is a thing smaller than a bird," or more briefly, "The ant is smaller than a bird." In the second judgment we assert an objective identity between "ant" and "thing smaller than a bird." "Smaller than a bird" is a relative attribute which is predicated of the ant.

Every judgment is preceded not only by two simple apprehensions by which the mind acquires the two ideas which constitute the matter of the judgment; it is also preceded by a comparative apprehension by which the mind apprehends (perceives) the objective identity or diversity of those two ideas. This comparative apprehension is called **Complex Apprehension.** If the mind, upon comparing together the formal objects of these ideas, does not apprehend their identity or diversity, then there is comparison, but no complex apprehension; and consequently, unless the mind is influenced by the will, there can be no judgment in regard to the objects as compared, but only a state of doubt.

20. An **à priori judgment** is a judgment which asserts an objective identity or diversity perceived by means of a mere comparison of the formal objects of two ideas either with each other or with the formal object of another idea; e.g. "The whole is greater than any of its parts;" "The square of the hypotenuse of a right-angled triangle is equal to the sum of the squares of the two other sides;" "The triangle is not round."

An **à posteriori judgment** is a judgment which asserts an objective identity or diversity perceived by

means of experience in addition to comparison; e.g.
"This iron is hot;" "The earth moves round the sun;"
"The moon is not self-luminous."

An *à priori* judgment is also called **analytical,** be-
cause by analysis of the subject we find the predicate
contained in it or excluded from it; **pure,** because we
find the identity or diversity of the formal objects of
the two ideas without recurring to experience; **neces-
sary,** because the identity or diversity is necessary and
cannot under any circumstances be lacking.

An *à posteriori* judgment, for opposite reasons, is
called **synthetic, experimental** (or **empirical), and con-
tingent.**

The matter of an *à priori* judgment is called **neces-
sary matter.** The matter of an *à posteriori* judgment
is called **contingent matter.**

An **immediate judgment** is a judgment which as-
serts an objective identity or diversity perceived with-
out recourse to proof; e.g. "The whole is greater than
any of its parts;" "The triangle is not round;" "This
iron is hot."

A **mediate judgment** is a judgment which asserts an
objective identity or diversity perceived by means of
proof; e.g. "The square of the hypotenuse of a right-
angled triangle is equal to the sum of the squares of
the two other sides;" "The earth moves round the
sun."

CHAPTER IV

THE CATEGORICAL PROPOSITION

21. The **categorical proposition** is the verbal expression of the matter and form of a judgment. It consists of two terms and the copula.

The *matter* of a judgment, that is, the formal objects of the two ideas which enter into it, is expressed by the two terms which stand in the positions of subject and predicate. Accordingly, the two terms which stand as subject and predicate are called the **matter** of the categorical proposition.

The *form* of a judgment, that is, the objective identity or diversity of the two ideas, is expressed by the copula. Hence, the copula is called the **form** of the categorical proposition. The **copula** is always "is" or "is not" or some other part of the present tense, indicative mood, of the verb "to be." Cf. Appendix: *Note on Section 21.*

Care must be taken not to confuse the logical predicate of a proposition with the *grammatical* predicate. The grammatical predicate is a finite verb; e.g. "The bird *sings."* Here the grammatical predicate is "sings." In order to throw this proposition into logical shape as the formal expression of the judgment, we must write it as follows: "The bird is singing" (cf. p. 46).

A categorical proposition is said to be in **logical form,** when it expresses formally and sepa-

rately the subject, the copula, and the predicate.

It will be well for the beginner in Logic to set down in logical form all the propositions with which he has to deal; repeated practice of this exercise will enable him to detect at once the matter and form of the judgments which are expressed by the propositions. Once this facility is acquired, there will be no need to insist upon the full logical form of the proposition, and he may say indifferently, even in argument, "The bird sings" or "The bird is singing."

For purposes of rhetorical effect, the predicate is sometimes written first; for example, "Great is Diana of the Ephesians." Again, we sometimes find a judgment expressed by a single word; for example, "Dinner!" which means "Dinner is prepared."

It is to be observed that a term, whether it be simple or complex (cf. 12), is nevertheless a single term. No matter what be the number of *words* in a complex term, the term itself is one and expresses only one idea. Hence, the subject, copula, and predicate of a proposition may consist of only three words, as in the proposition, "Man is rational," or they may consist of many more, as in the proposition, "The learned economist who was here last Saturday has met with a serious accident."

QUALITY OF CATEGORICAL PROPOSITIONS

22. The **quality** of a categorical proposition is the character of the copula.

An **affirmative proposition** is a categorical proposition in which the copula consists either formally or

elliptically of the verb "is;" e.g. "Gold is yellow;" "The King rules England." The affirmative proposition is the verbal expression of the matter and form of an affirmative judgment.

A **negative proposition** is a categorical proposition in which the copula consists either formally or elliptically of the verb "is" modified by a negative particle; e.g. "A feather is not heavy;" "The horse does not fly." The negative proposition is the verbal expression of the matter and form of a negative judgment.

If the negative particle, instead of modifying the copula, immediately modifies the subject or the predicate, the proposition is affirmative; e.g. "These men are non-combatants;" "Non-members oppose the organization;" "Those who are not of age are exempt."

In an *affirmative* proposition the subject and predicate stand for the same thing, but in a different way; that is, the subject stands for the thing *which* has the form signified by the predicate, and the predicate stands for the thing *as* having this form.

In a *negative* proposition the subject and predicate stand for two different or distinct things.

QUANTITY OF CATEGORICAL PROPOSITIONS

23. The **quantity of a term** is the completeness with which the term refers severally to the individuals comprised in the extension of the universal idea expressed by it. According as the term refers severally, on the one hand, to all the objects, or, on the other, to an indeterminate object or an indeterminate number of the objects in the extension of a universal idea,

it is said to be **distributed** or **undistributed** (cf. 16, 8).

The **quantity of a categorical proposition** is the quantity of the subject-term.

A **universal proposition** is a categorical proposition which has a distributed term for its subject; that is, it is a categorical proposition in which the predicate is affirmed or denied of each and every individual comprised under the extension of the subject-idea; e.g. "Every just man is deserving of praise;" "No horse is rational."

N. B.—A proposition is called universal, not only when the subject is a distributed term, but often also when the subject is a general term which is used absolutely and the predicate, too, is a general term (cf. 18) ; e.g. "Man is mortal;" "The circle is round." It is plain that the subject of each of these propositions can be changed into a distributed term on demand. The reason is, that "man" and "circle" are formal objects of universal ideas (cf. 8), and hence, what is true of "man" and "circle" without qualification or restriction is true of "every man" and "every circle."

A **particular proposition** is a categorical proposition which has an undistributed term for its subject; that is, it is a categorical proposition in which the predicate is affirmed or denied severally of an indeterminate individual or an indeterminate number of the individuals comprised under the extension of the subject-idea; e.g. "Some men are truthful;" "Some men are not strong." The particular proposition is also called an **Indefinite Proposition**.

A **singular proposition** is a categorical proposition which has a singular term for its subject; that is, it is

a categorical proposition in which the predicate is affirmed or denied of one determinate individual or of certain determinate individuals; e.g. "Cicero is famous;" "This dog is mad;" "These stones are not valuable."

An **indesignate proposition** is a categorical proposition which has for its subject a general term with no sign of quantity accompanying it; e.g. "Man is rational;" "Frenchmen are polite;" "Crows are not white."

An **absolutely universal proposition** is a proposition which states something that holds good of the full extension of the subject-idea; e.g. "Every circle is round;" "No triangle is four-sided."

A **morally universal proposition** is a proposition which states something that holds good of only the greater part of the extension of the subject-idea; e.g. "All old people praise past times;" "No mother hates her child."

For the logician a morally universal proposition is a *particular* proposition. When he speaks of universal propositions, he means propositions which are *absolutely* universal.

24. Whether an indesignate proposition is absolutely or morally universal is to be determined by the judgment which it is intended to express. When an indesignate proposition, which is only morally universal, is put forth in argument as absolutely universal, it is to be denied.

When the subject-term is used collectively, that is, when we have collective *supposition*, the proposition is frequently, though by no means always, equivalent to a

singular proposition; e.g. "All the Apostles are twelve."
  When the subject of a proposition is a collective
term, the proposition may be either universal, particu-
lar, or singular.

So far as its use in argument goes, the singular
proposition is equivalent to a universal proposition;
because in it the predicate is affirmed or denied of the
full extension of the subject-idea, though this exten-
sion is only one individual or certain definite indi-
viduals.

25. **Signs of Quantity.** In an *affirmative* proposition
the sign "every" or "each," prefixed to the subject,
makes the proposition universal. The sign "all"
generally does so, but not always; for it is some-
times used in collective supposition; e.g. "All the
angles of a triangle are equal to two right angles"
(cf. 18). The sign "every" should be used instead of
"all" when the latter leaves an opening for ambiguity.
In Logic we shall, for the sake of convenience, employ
the formula "All S is P" for the universal affirmative
proposition.

In a *negative* proposition "every," "each," and "all,"
prefixed to the subject, are equivalent to the sign
"some," and they make the proposition particular;
e.g. "All men are not wealthy" or "Not all men are
wealthy" == "Some men are not wealthy."

• The usual sign of universality in a negative prop-
osition is "no" prefixed to the subject; e.g. "No horse is
rational." This does not conflict with the definition
of negative proposition in section 22; for this proposi-
tion is the same as the following: "Horses are not
rational."

"Some" is the sign usually prefixed to the subject to make the proposition particular. In Logic "some" has the force of "one at least" or "a certain (or uncertain) number of." Hence, "some" in Logic is not inconsistent with "all;" for example, the following propositions are not inconsistent with each other: "All men are mortal," "Some men are mortal."

A proposition beginning with "whoever," "whatever," "he who," "those who," "that which," or an equivalent expression, is universal.

The sign "a few" makes the proposition particular, but does not of itself affect the quality of the proposition.

The sign "few" commonly has the same force as "only a few;" and the proposition in which it occurs can usually be resolved into two propositions, one affirmative, and the other negative; e.g. "Few lawyers achieve world-wide fame" = "A few (or some) lawyers achieve world-wide fame" and "Most (or more) lawyers do not achieve world-wide fame." What has been said of "few" is also true of the signs "hardly any" and "scarcely any."

N. B.—The sign of quantity is not part of the term.

26. **Relation of the Comprehension of the Predicate to that of the Subject.** In an *affirmative* proposition we assert that all the notes in the explicit and implicit comprehension of the predicate, whether taken *separately* or *collectively,* are found in the object denoted by the subject. The reason is, that in an affirmative proposition we assert that the object denoted by the subject *is* (that is, is *identical* with) the object denoted by the predicate, and

hence, the object denoted by the subject has each and all the notes in the explicit and implicit comprehension of the predicate. For example, "Man is an animal:" "man" has each and all the notes that make up the explicit and implicit comprehension of "animal," e.g. sentient, organic, corporeal, substance.

In a *negative* proposition we assert that the notes in the comprehension of the predicate, taken *collectively*, are not found in the object denoted by the subject; but we do not assert that no portion of those notes is found in that object. The reason is, that in a negative proposition we assert that the objects denoted by the subject and predicate are different or distinct things; and the possession by the predicate-object of one note which is not found in the object denoted by the subject constitutes the predicate-object a *different* thing from the object denoted by the subject. When we say "A man is not a horse," we mean the *collection* of notes which make up the comprehension of "horse" (e.g. equine, sentient, organic, corporeal, substance) is not found in "man;" we do not mean that he possesses none of these notes.

27. A natural proposition is either an affirmative proposition in which the idea expressed by the predicate has an extension at least as wide as that of the idea expressed by the subject, or it is a negative proposition; e.g. "All men are mortal;" "All men are rational animals;" "Some men are white;" "Some men are not lawyers."

An **unnatural proposition** is an affirmative proposition in which the idea expressed by the predicate has a narrower extension than that of the idea expressed

by the subject; e.g. "Some mortal beings are men;" "Some animals are horses."

28. In Logic the two first vowels of the Latin word "affirmo" (= "I affirm") are used to denote respectively the universal affirmative and the particular affirmative proposition; the two vowels of the word "nego"(= "I deny") denote respectively the universal negative and the particular negative proposition. Thus, we have the four typical propositions symbolized as follows:

> A = Universal affirmative
> E = Universal negative
> I = Particular affirmative
> O = Particular negative

DISTRIBUTION OF THE PREDICATE

29. In works on Logic the predicate of an affirmative proposition is usually set down as undistributed, and the predicate of a negative proposition, as distributed. Thus, according to this doctrine we should have the following table:

In A,	the subject is	distributed,	the predicate,	undistributed
In E,	" " "	distributed,	" "	distributed
In I,	" " "	undistributed,	" "	undistributed
In O,	" " "	undistributed,	" "	distributed

The hypothesis of the distribution of the predicate was devised in order to simplify the explanation of Conversion and of the Categorical Syllogism. But when we come to the Categorical Syllogism, we shall find that the explanation is much easier and simpler if this hypothesis is discarded. Our main reason for

not adopting it will be explained in the Appendix (cf. *Note on Section 29*). For the present, we shall attempt to show, without recourse to this hypothesis, where the process of Conversion is possible and what its result is in the case of each proposition.

30. **Conversion of Affirmative Propositions.** In an *affirmative* proposition we assert that the object denoted by the subject and the object denoted by the predicate are one and the same thing, that the subject-object has the attribute signified by the predicate; but we do not advert to the question whether there are any other objects which have this attribute.   Whether any object besides the subject-object possesses this attribute, we cannot learn from the proposition; to find this out, we must appeal to some other source of information.   "Every man is an animal," "Every man is a rational animal:" — prior or subsequent to the utterance of these propositions we may know whether or not "animal" and "rational animal" are co-extensive in application with "man;" but the proposition itself does not decide the question for us.   Hence, if we wish to make the subject and predicate change places, it is not the proposition itself, but information secured elsewhere, which will determine what sign of quantity we are *objectively* warranted in prefixing to the proposition.   Since "Every man is an animal," we know by *reflection* that there are at least as many objects with the attribute signified by "animal" as there are men, and that each of these objects has the attribute signified by "man;" hence, we are warranted in predicating "man" of each of these objects.   But since we do not know from the proposition

whether or not there are any objects besides men
which possess the attribute signified by "animal," we
cannot say "Every animal is a man," but only "Some
animals are men."

What has been said of the proposition, "Every man
is an animal," can be applied *mutatis mutandis* to the
particular affirmative proposition, "Some horses are
black." Our rule, then, for the conversion of affirma-
tive propositions is that the subject of the converse
must have the particular sign of quantity. Therefore,
the converse of "All S is P" and "Some S is P" is
"Some P is S."

31. **Conversion of Negative Propositions.** In a *nega-
tive* proposition we assert that the object denoted by
the subject and the object denoted by the predicate
are different (distinct) things, that the subject-
object lacks the attribute signified by the predicate.
In the *universal* negative proposition, "No horse
is rational," we assert that the horse lacks the
attribute "rational." Hence, by *reflection* we know
that any object with the attribute "rational" lacks the
attribute signified by "horse." Consequently, if we
wish to make the subject and predicate change places,
we are warranted by information furnished by the
proposition itself in prefixing the universal sign of
quantity to the converse; thus, "No rational being is
a horse." For this reason, the proposition, "No S is
P," may always be changed into "No P is S."

The *particular* negative proposition, "Some swans
are not white," cannot be changed into "No white
things are swans;" for this could again be changed

into "No swans are white," which asserts more than the original proposition, and hence is not one of its implications. It cannot be changed into "Some white things are not swans;" for this supposes information as to the number of white things which cannot be obtained from the original proposition: so far as we know from the original proposition, all the white things may be swans. This would be still more evident, if we attempted to convert the proposition, "Some animals are not horses." Hence, the particular negative proposition, "Some S is not P," cannot be converted.

COMPOUND CATEGORICAL PROPOSITIONS

32. A **simple categorical proposition** is a proposition which expresses the matter and form of only one judgment, and hence has only one subject and one predicate.

A **compound categorical proposition** is a proposition which expresses the matter and form of two or more judgments, and hence has several subjects or several predicates. It is either formal or elliptical.

33. A **formal compound categorical proposition** is a proposition in which the several judgments expressed are evident from the structure of the proposition.

A **conjunctive categorical proposition** is a proposition in which there are several subjects or several predicates connected by the particle "and" or an equivalent, expressed or understood; e.g. "All lawyers and physicians are professional men." This proposition is equivalent to the two following simple propositions:

"All lawyers are professional men," "All physicians are professional men." The foregoing compound proposition may also be written in the form, "Any one who is either a lawyer or a physician is a professional man."

A **remotive proposition** is a negative proposition in which there are several subjects or several predicates connected by "neither . . . nor" or equivalent particles; e.g. "The horse is neither horned nor cloven-footed" = "The horse is not horned" and "The horse is not cloven-footed;" "Neither aliens nor minors nor criminals are voters" = "Aliens are not voters," "Minors are not voters," and "Criminals are not voters." These two examples may also be written as follows: "No horse is either horned or cloven-footed," "No one who is either an alien, or a minor, or a criminal is a voter."

An **adversative categorical proposition** is a proposition in which there are several subjects or several predicates connected by the particle "but" or an equivalent; e.g. "Cicero was not a great general, but a great orator; "Not Paul, but Peter, was Bishop of Rome."

34. An **elliptical compound categorical proposition** is a proposition in which the several judgments expressed are not evident from the structure of the proposition, but become evident when the proposition is resolved. This kind of proposition is also called an **Exponible Proposition.**

An **exclusive proposition** is a proposition which, by means of a word such as "only," "alone," "none but," excludes what is asserted from everything except the subject; e.g. "Only graduates are eligible" (or "Graduates are the only eligible persons") ; "None but Seniors

were present." The principal judgment expressed by these propositions is generally universal negative, viz. "No non-graduates are eligible;" "No non-Seniors were present." The other judgment is affirmative, usually particular, viz. "Some graduates are eligible;" "Some Seniors were present."

An **exceptive proposition** is a proposition which, by means of a word like "except", excludes what is asserted from one or more of the inferiors of the subject-idea (cf. 3, 8); e.g. "All animals except men are irrational" = "Men are not irrational," "All other animals are irrational."

A **comparative proposition** is a proposition which affirms or denies that an attribute belongs to the subject in the same degree as it does to something else or in a greater or a less degree; e.g. "Philosophy is more important than eloquence." This proposition, in addition to the judgment formally expressed, involves the two following judgments: "Philosophy is important," "Eloquence is important."

An **inceptive proposition** is a proposition which makes an assertion as to the commencement of something; e.g. "Wilson became President on March 4, 1913." This proposition is equivalent to the following: "Wilson was not President before March 4, 1913" and "Wilson was President on and after March 4, 1913."

A **desitive proposition** is a proposition which makes an assertion as to the ending of something: e.g. "Taft ceased to be President on March 4, 1913." This proposition is equivalent to the following: "Taft was President on and before March 4, 1913" and "Taft was not President after March 4, 1913."

Reduplicative and **specificative propositions** are propositions which contain an iterative particle such as "inasmuch as," "in so far as," "as such," or "as," the force of which is to indicate in what way or in what sense the predicate belongs to the subject. If the particle indicates the form according to which the predicate *must* belong to the subject, the proposition is **reduplicative**; e.g. "Man, inasmuch as he is intelligent, is free" = "Man, *because* he is intelligent, is free." If the particle merely indicates the form according to which the predicate *does* or *can* belong to the subject, the proposition is **specificative**; e.g. "The physician, so far as he is a man, reads and speaks" = "The physician reads and speaks, and the form according to which he does or can read and speak is his humanity."

MODAL CATEGORICAL PROPOSITIONS

35. An **assertoric categorical proposition** is a proposition which asserts the *fact* of the objective identity or diversity of two ideas. The propositions we have been dealing with thus far are mostly assertoric.

A **modal categorical proposition** is a proposition which asserts the *necessity* or the *possibility* of the objective identity or diversity of two given ideas.

Modal propositions are divided into apodeictic and problematic.

An **apodeictic proposition** is a modal proposition which asserts the *necessity* of the objective identity or diversity of two given ideas; e.g. "That a circle should be round is necessary;" "That a square should

not be round is necessary." To say that the objective diversity of two ideas is necessary is equivalent to saying that the objective *identity* of those two ideas is *impossible.* For this reason, the second of the two examples is usually written, "That a square should be round is impossible."

The matter of an apodeictic proposition is called **necessary matter.**

A **problematic proposition** is a modal proposition which asserts the *possibility* of the objective identity or diversity of two given ideas; e.g. "That a king should be wise is possible;" "That the weather should not be fair is possible." The word "contingent" is used in Logic in the sense of "not necessary." To say that the objective diversity of two ideas is possible is equivalent to saying that the objective *identity* of those two ideas is *contingent.* For this reason, it is customary to write the second of the two examples as follows: "That the weather should be fair is contingent."

The matter of a problematic proposition is called **contingent matter.**

The objective *identity* of the two given ideas, whether it be necessary, impossible, possible, or contingent, is called the **dictum,** when expressed as the subject of the modal proposition; the predicate is called the **mode.** Since the predicate is one or other of the four terms, "necessary," "impossible," "possible," "contingent," there are **four modes.**

Since the modal proposition makes an assertion about the objective identity or diversity of two given ideas, and since the objective identity or diversity is

expressed by the copula, the mode may be expressed by a verb which immediately modifies the copula. Thus, the four propositions we have employed in illustration may be worded as follows: "A circle *must* be round" (**necessary**); "A square *cannot* be round" (**impossible**); "A king *may* be wise" (**possible**); "The weather *need not* be fair" (**contingent**).

OPPOSITE CATEGORICAL PROPOSITIONS

36. Opposite categorical propositions are categorical propositions which are identical in matter but different in form. They are illustrated in the following **Square of Opposition**:
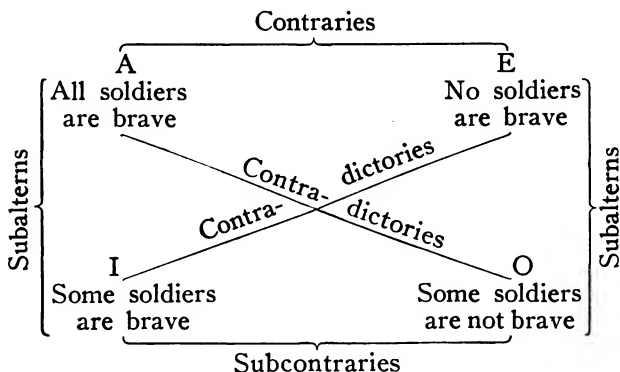
Contradictory propositions are two opposite propositions one of which is universal and the other particular; e.g. "All soldiers are brave"—"Some soldiers are not brave;" "No soldiers are brave"—"Some soldiers are brave." In each of these pairs one proposi-

tion affirms or denies just enough to make the other false. Contradictory propositions cannot both be true nor both be false at the same time; but one is true and the other false. Hence, from the truth of one we can infer the falsity of the other; and from the falsity of one we can infer the truth of the other.

In order to disprove a given proposition, it is necessary and sufficient to prove its contradictory.

Contrary propositions are two opposite propositions both of which are universal; e.g. "All soldiers are brave"—"No soldiers are brave." One of the propositions affirms or denies more than is necessary to make the other false. Contrary propositions cannot both be true, but both may be false, at the same time. Hence, from the truth of one we may infer the falsity of the other; but from the falsity of one we cannot infer the truth of the other.

To refute a universal proposition, it is not necessary to prove its contrary; but it is frequently of great advantage, if it can be done; for then the refutation is overwhelming and manifest to everyone.

Subcontrary propositions are two opposite propositions both of which are particular; e.g. "Some soldiers are brave"—"Some soldiers are not brave." Subcontrary propositions cannot both be false, but both may be true, at the same time. If they were in any case false together, then their respective contradictories would have to be true together, and such propositions as the following would both be true: "All soldiers are brave," "No soldiers are brave;" but this is impossible.

When it is said that subcontraries may be true together, the meaning is that, unless we have evidence that the contradictory of one of them is true, we have no right to assume that the subcontraries are not both true.

Subaltern propositions are two propositions identical in matter and form, but different in quantity; e.g. "All soldiers are brave"—"Some soldiers are brave;" "No soldiers are brave"—"Some soldiers are not brave." Subaltern propositions may both be true and may both be false at the same time. From the truth of the universal we may infer the truth of the particular, but not *vice versa*. From the falsity of the particular we may infer the falsity of the universal, but not *vice versa*.

The following table indicates the inferences which may be drawn from the truth or falsity of the propositions A, E, I, O, in terms of each other:

If A is true,	E is false,	I is true,	O is false
If A is false,	E is unknown,	I is unknown,	O is true
If E is true,	A is false,	I is false,	O is true
If E is false,	A is unknown,	I is true,	O is unknown
If I is true,	A is unknown,	E is false,	O is unknown
If I is false,	A is false,	E is true,	O is true
If O is true,	A is false,	E is unknown,	I is unknown
If O is false,	A is true,	E is false,	I is true

The definitions and remarks in the preceding paragraphs of this section are based on the supposition that we are dealing exclusively with the four typical forms of the categorical proposition. In order to extend the doctrine of Opposition so that it shall apply to all kinds of propositions, we shall have to employ the definitions which are given below in section 48.

The opposition between **modal propositions** is brought out in the following diagram:

Contraries

A
Iron must
be solid

E
Iron cannot
be solid

Subalterns

Subalterns

Contra- dictories

Contra- dictories

I
Iron may
be solid

O
Iron need not
be solid

Subcontraries
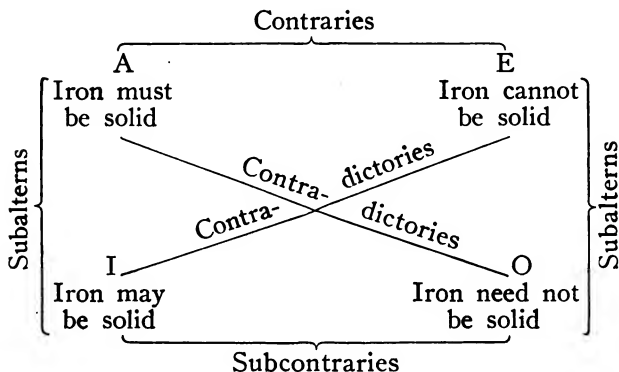
A **singular proposition,** like "Plato is clear," is vague, and it would be difficult to say offhand whether "Plato is not clear" is its contradictory or its contrary. When a proposition of this kind is qualified by some word or expression, such as "always," "everywhere," "partly," "in some places," or "in some respects," it is possible to employ it as a basis for a square of opposites.   Thus:

1. Plato is always clear.
2. Plato is never clear.
3. Plato is sometimes clear.
4. Plato is sometimes not clear.

1 and 4 and also 2 and 3 are **contradictories**; 1 and 2 are **contraries**; 3 and 4 are **subcontraries**; 1 and 3 and also 2 and 4 are **subalterns**.

CHAPTER V

EDUCTION

37. The **import** or **meaning** of a categorical proposition is that which is explicitly asserted by the judgment which it expresses; it is that which is adverted to in the act of judging. In other words, it is the formal object of the judgment.

The **implication** of a categorical proposition is a judgment or several judgments involved in the import of the proposition. Thus, the judgment expressed by the proposition, "All men are rational," involves the judgment, "No irrational beings are men." The implications of a given judgment are all the additional judgments to which a man necessarily commits himself in pronouncing that judgment, though he may not have actually formulated these other judgments in his mind.

Eduction is the process of drawing out the implication of a single proposition. There are four kinds of eduction which have a prominent place in works on Logic, viz. Conversion, Obversion, Contraposition, and Inversion.

As there will be frequent mention of positive and negative terms in the next few sections, a word should be said in explanation of them.

A **positive term** is a term which expresses a positive idea (cf. 9) ; that is, it is a term which denotes a form

or an object *as* possessing that form; e.g. "rational," "rational being."

A **negative term** is a term which expresses a negative idea; that is, it is a term which denotes the absence of a form or an object *as* lacking that form; e.g. "irrational," "irrational being," "non-metallic."

Positive terms we shall symbolize by the letters S and P (S standing for subject, and P for predicate), and negative terms, by non-S and non-P.

The positive and negative terms of which we speak in eduction are contradictory of each other; that is, they are such that any pair of them comprises all objects whatever (cf. 9). Care must, therefore, be taken that the negative term which is employed shall be the *contradictory,* and not the *contrary,* of the positive term. Almost always the contradictory of a *simple* term may be obtained by prefixing "non" to it. Other negative prefixes and suffixes frequently have this force, but more often not. "Invisible," "irrational," "untainted," "useless," have the same meaning respectively as "non-visible," "non-rational," "non-tainted," "nonuseful;" but "unpleasant," "unholy," "immoral," "discourteous," are not the same as "non-pleasant," "nonholy," "non-moral," "non-courteous." We may speak, for instance, of a lump of coal as *non-holy,* but we cannot speak of it as *unholy.*

S means "thing (or being) that is S," and non-S, "thing (or being) that is not S;" P means "thing (or being) that is P," and non-P, "thing (or being) that is not P." Thus, the Partial Inverse of "Every truthful man is mortal" is "Some beings that are not truthful men are not mortal." "Non-truthful man" is not

the contradictory of "truthful man;" for the two terms, "truthful man" and "non-truthful man," do not comprise between them all objects whatever.

We have seen that a pair of contradictory terms, such as S—non-S, P—non-P, comprises all objects whatever. If in any case the one term S (or P) extends by itself to all objects, then there is no non-S (or non-P); that is, there is no "thing that is not S" (or "thing that is not P"). "Thing," for instance, extends by itself to all objects whatever, and therefore there is no object which can be denoted by the negative of "thing;" that is, there is no "thing that is not a thing."

When non-S is lacking, any eduction which involves the existence of non-S is impossible; the same is true *mutatis mutandis* when non-P is lacking, and again when S is lacking, and still again when P is lacking.

CONVERSION.

38. **Conversion** is an eduction by which from a given proposition another is derived having for its subject the original predicate and for its predicate the original subject.

The original proposition is called the **Convertend**, and the derived proposition, the **Converse**.

RULE for conversion: *The quality of the converse must be the same as that of the convertend.*

The reason for the rule is that an identity or a diversity remains an identity or a diversity (cf. 19), whether we view it from the standpoint of the subject or from the standpoint of the predicate.

A second rule is usually laid down in works on Logic as follows: *In the converse no term may be distributed which was not distributed in the convertend.* But this rule assumes that the predicate of a categorical proposition is distributed or undistributed. This assumption we have seen to be unnecessary, and hence there is no need for the rule (cf. 29, 30, 31).

Simple conversion is a conversion in which the converse has the same quantity as the convertend.

Conversion per accidens is a conversion in which a particular converse is derived from a universal convertend.

E and I can be converted *simply:*

E No trees are sentient—No sentient things are trees.
I Some flowers are fragrant—Some fragrant things are flowers.

A can be converted only *per accidens;* E *may* be converted *per accidens:*

A All men are mortal—Some mortal beings are men.
E No trees are sentient—Some sentient things are not trees.

O cannot be converted.

The justification of these statements will be found in sections 30 and 31.

The results we have reached in this section are exhibited in the following table:

Convertend	Converse
A All S is P	Some P is S
E No S is P	No P is S
I Some S is P	Some P is S
O Some S is not P	. . . .

Before a proposition is converted, it should be thrown into logical form; that is, the subject, the copula, and the predicate should be set down definitely and distinctly. "A stitch in time saves nine:"—in logical form this proposition will read "A stitch in time is a thing that saves nine stitches;" and its converse is "Something that saves nine stitches is a stitch in time."

Moreover, unless the *matter* of the proposition makes it imperative, we must be careful not to change the predicate into a singular term by the use of the definite article "the;" for example, the logical form of "Peter struck James" is not "Peter is the person who struck James," but "Peter is a person who struck James," and the converse is "Some one who struck James is Peter."

Sometimes, however, the matter of the proposition makes it necessary to use the definite article before the predicate, because the predicate is seen from the matter to be a singular term. Thus, the proposition, "Dickens wrote David Copperfield," in logical form is "Dickens is the person who wrote David Copperfield," from which we derive the converse, "The person who wrote David Copperfield is Dickens."

Again, when casting a proposition into logical form, we should not place at the beginning of the predicate such words as "he who," "that which," "those who," "those which," "the persons who," "the things which,"—unless, indeed, the *matter* evidently calls for them. The insertion of such words in the predicate will generally make the new form of the proposition say more than the original form.

OBVERSION

39. Obversion is an eduction by which from a given proposition another is derived having for its subject the original subject and for its predicate the contradictory of the original predicate.

The original proposition is called the **Obvertend,** and the derived proposition, the **Obverse.**

RULE: *To obtain the obverse, negative the predicate and change the quality, but not the quantity, of the obvertend.*

Examples of obversion:

Obvertend	Obverse
A All metals are material—No metals are non-material (immaterial).	
E No horses are rational—All horses are non-rational (irrational).	
I Some men are tactful—Some men are not. non-tactful. (tactless).	
O Some substances are not visible—Some substances are non-visible (invisible).	

A obverts to E; E to A; I to O; O to I.
We have then the following table:

Obvertend	Obverse
A All S is P	No S is non-P
E No S is P	All S is non-P
I Some S is P	Some S is not non-P
O Some S is not P	Some S is non-P

The process of obversion is justified as follows:

When the subject-object is identical with the predi-
cate-object (*All S is P, Some S is P*), it is really dis-
tinct from anything that is really distinct from the
predicate-object (*No S is non-P, Some S is not non-P*)
(cf. 19, 4). When the subject-object is really distinct
from the predicate-object (*No S is P, Some S is not P*),
it is identical with something that is really distinct
from the predicate-object (*All S is non-P, Some S is
non-P*); that is, it is identical with itself, and this is
all the information the proposition gives us beyond the
fact that the subject-object is really distinct from the
predicate-object. *No S is non-P* and *All S is non-P* may
be expressed respectively as follows: *S is really distinct
from any object which is really distinct from P; S is iden-
tical with some object which is really distinct from P.*

We justify the process of obversion, as expressed in
language, by saying that two negatives are equivalent
to an affirmative.

CONTRAPOSITION

40. **Contraposition** is an eduction by which from a
given proposition another is derived having for its
subject the contradictory of the original predicate and
for its predicate the contradictory of the original
subject.

The original proposition we shall call the **Contra-
ponend**; the derived proposition is called the **Contra-
positive.**

When the derived proposition has for its predicate
the *subject* of the original proposition, and not its con-
tradictory, it is called the **Partial Contrapositive.**

RULE: *To obtain the partial contrapositive, obvert the original proposition and then convert the obverse.*

To obtain the contrapositive, obvert the partial contrapositive.

A, E, and O may be contraposited.

I cannot be contraposited; because, after being obverted, it becomes O, and O cannot be converted.

Examples of contraposition:

A

Contraponend	All residents are combatants.
Obverse	No residents are non-combatants.
Partial Contrapositive	No non-combatants are residents.
Contrapositive	All non-combatants are non-residents.

The partial contrapositive of A may also be written in the form, "Only combatants are residents" or "None but combatants are residents."

E

Contraponend	No professionals are members.
Obverse	All professionals are non-members.
Partial Contrapositive	Some non-members are professionals.
Contrapositive	Some non-members are not non-professionals.

O

Contraponend	Some Americans are not voters.
Obverse	Some Americans are non-voters.
Partial Contrapositive	Some non-voters are Americans.
Contrapositive	Some non-voters are not non-Americans.

The following table sums up these results in symbolical form:

Contraponend	Partial Contrapositive	Contrapositive
A All S is P	No non-P is S	All non-P is non-S
E No S is P	Some non-P is S	Some non-P is not non-S
I Some S is P	. . . .	. . . .
O Some S is not P	Some non-P is S	Some non-P is not non-S

It is to be observed that the contrapositive is of the same quality as the contraponend, whereas the partial contrapositive is of opposite quality. Again, the partial contrapositive and the contrapositive of *All S is P,* being both universal, allow us to pass back from them to the original proposition, *All S is P.* But this reverse process is not possible in the case of the proposition *No S is P,* because its partial contrapositive and its contrapositive are both particular.

When "No non-P is S" is derived from "All S is P," it is called the partial contrapositive of "All S is P." When "All S is P" is derived from "No non-P is S," it is called the **obverted converse** of "No non-P is S." Thus, the obverted converse of "No beings that are not mortal are men" is "All men are mortal."

INVERSION

41. Inversion is an eduction by which from a given proposition another is derived having for its subject the contradictory of the original subject and for its predicate the contradictory of the original predicate.

The original proposition is called the **Invertend,** and the derived proposition, the **Inverse.**

When the derived proposition has for its predicate the *predicate* of the original proposition, and not its contradictory, it is called the **Partial Inverse.**

RULE: *To obtain the inverse of A, obvert and convert alternately through four steps.*

To obtain the inverse of E, convert and obvert alternately through four steps.

To obtain the partial inverse of A and E, obvert the inverse of A and E respectively.

A and E may be inverted.

I and O cannot be inverted; for if we attempt to invert either of them, we shall be confronted in the process by an O proposition to be converted, and O does not admit of conversion.

Examples of inversion:

A

Invertend	All residents are combatants.
By obversion	No residents are non-combatants.
By conversion	No non-combatants are residents.
By obversion	All non-combatants are non-residents.
By conversion	Some non-residents are non-combatants.

The proposition "Some non-residents are non-combatants" is the **inverse** of "All residents are combatants." By obverting the inverse we obtain the **partial inverse,** viz. "Some non-residents are not combatants."

E

Invertend	No professionals are members.
By conversion	No members are professionals.
By obversion	All members are non-professionals.
By conversion	Some non-professionals are members.
By obversion	Some non-professionals are not non-members.

"Some non-professionals are not non-members" is the **inverse** of "No professionals are members." By obverting the inverse we obtain the **partial inverse,** viz. "Some non-professionals are members."

The inverse is of the same quality as the original proposition, but the partial inverse is of opposite quality. The quantity, whether of the inverse or of the partial inverse, is always particular.

Since only universal propositions can be inverted, we may lay down the following simple rule for inversion:

RULE FOR THE INVERSE: *Replace the subject and the predicate of the invertend by their respective contradictories and change the quantity from universal to particular.*

The following table shows the results we have reached:

Invertend	Partial Inverse	Inverse
A All S is P	Some non-S is not P	Some non-S is non-P
E No S is P	Some non-S is P	Some non-S is not non-P

Since the letters A, E, I, O represent the four typical forms of the categorical proposition, and since S and P stand for subject and predicate respectively, we may represent the four propositions concisely by the symbols SaP, SeP, SiP, SoP. Letting S' and P' denote non-S and non-P respectively, we may bring together in the following table the results of the four processes of eduction we have thus far considered:

	A	E	I	O
1. Original proposition	SaP	SeP	SiP	SoP
2. Obverse	SeP'	SaP'	SoP'	SiP'
3. Converse	PiS	PeS	PiS	
4. Obverted converse	PoS'	PaS'	PoS'	
5. Partial contrapositive	P'eS	P'iS		P'iS
6. Contrapositive	P'aS'	P'oS'		P'oS'
7. Partial inverse	S'oP	S'iP		
8. Inverse	S'iP'	S'oP'		

OTHER FORMS OF EDUCTION

42. Eduction by an added determinant is an eduction by which from a given proposition another is derived in which the original subject and predicate are qualified by the same incident term (cf. 12). For example, "All heroes are benefactors"—"All American heroes are American benefactors."

It is to be observed that the *same* incident term must qualify the subject and the predicate of the derived proposition. If the term has a different meaning or force in the subject from what it has in the predicate, the eduction is not valid. For example, "A ballplayer is a man"—"A poor ball-player is a poor man."

Quantitative terms, like "small" and "large," are espe-
cially apt to lead to fallacy; e.g. "An ostrich is a bird"
—"A small ostrich is a small bird."

43. Eduction by an omitted determinant is an educ-
tion by which from an *affirmative* proposition in
which the predicate contains an incident term is de-
rived another proposition in which the predicate lacks
the incident term (cf. 12). For example, "These men
are famous lawyers"—"These men are lawyers."

If the incident term is such as to change the mean-
ing of the principal term, this eduction is impossible;
e.g. "What this book relates are imaginary facts"—
"What this book relates are facts."

44. Eduction by complex conception is an eduction
by which from a given proposition another is derived
having the same relation added to the original subject
and predicate. For example, "A lark is a bird"—"The
feathers of a lark are the feathers of a bird;" "Gold is
a valuable metal"—"A ring made of gold is a ring made
of a valuable metal," or "Anything made of gold is (a
thing) made of a valuable metal."

It is to be observed, however, that there is danger
of committing a fallacy, if quantitative relations are
employed; e.g. "Physicians are professional men"—
"A majority of physicians are a majority of profes-
sional men;" "A carpenter is a mechanic"—"The most
skilful body of carpenters is the most skilful body of
mechanics." This kind of eduction from *negative*
propositions is apt to be invalid, if the relation em-
ployed is not a relation of part to whole; e.g. "A dog
is not a horse"—"The color of a dog is not the color

of a horse," or "The owner of a dog is not the owner
of a horse."

45. **Eduction by converse relation** is an eduction by
which from a given proposition, in which a relation
of the subject to another object is affirmed or denied,
is derived another proposition, in which the reverse
relation of the other object to the original subject is
affirmed or denied respectively. For example, "The ele-
phant is larger than a horse"—"The horse is smaller than
an elephant;" "John is the brother of Mary"—"Mary is
the sister of John;" "Washington is north of New
Orleans"—"New Orleans is south of Washington;" "A
is equal to B"—"B is equal to A;" "Robert is not the
father of James"—"James is not the son of Robert."

Akin to eduction by converse relation are such educ-
tions as the following: "The horse is larger than a
dog"—"Whatever is larger than a horse is larger than
a dog," "Whatever is as large as a horse is larger than
a dog," "Whatever is smaller than a dog is smaller
than a horse," "Whatever is as small as a dog is
smaller than a horse," "Whatever is as small as a dog
is not as large as a horse," etc.

Note.—The various processes we have considered
under the general head of Eduction are usually called
Immediate Inferences. But this name is misleading;
for it confuses the *act* with the *process* of inference.
(cf. 47). Cf. APPENDIX: *Note on Section 45.*

We said above that the name "Eduction" is applied
to the process of drawing out the implication of a
single proposition. This name might also be applied
to processes like the following: "All men are mortal;

All men are rational; Therefore all men are mortal
and rational;" "Therefore no being that is either im-
mortal or irrational is a man" (cf. 78).   But it will be
observed that in both cases the conclusion is a com-
pound categorical proposition.   .

CHAPTER VI

THE ACT OF INFERENCE

46. Inference is either an act or a process.

The **act of inference,** primarily, is an act of the mind asserting that in the world of reality the formal object of one potential judgment is logically dependent upon the formal object of one or more potential judgments (cf. 19). More briefly, but less accurately, the act of inference, primarily, is the mental assertion of the logical dependence of one potential judgment upon one or more potential judgments.

A **potential judgment** is a judgment which is not pronounced, but which *can be* pronounced. The **formal object** of a potential judgment is a formal object which is not asserted, but which *can be* asserted, by a judgment.

For the sake of brevity, we shall call the act of inference by the simple name of Inference.

The word **"Reasoning"** is a synonym for Inference.

The **formal object** or **form** of an inference, primarily, is the logical dependence of one potential judgment upon one or more potential judgments; or rather, it is the logical dependence of the formal object of one potential judgment upon the formal object of one or more potential judgments (cf. 19, 2).

Logical dependence is also called **Sequence.**

The **material object** or **matter** of an inference,

57

primarily, are two or more potential judgments, or
rather, the formal objects of two or more potential
judgments.

The **act of inference,** secondarily, is an act of the
mind asserting that in the world of reality the formal
object of one potential inference is logically dependent
upon the formal object of one or more potential in-
ferences; or more briefly, but less accurately, it is the
mental assertion of the logical dependence of one
potential inference upon one or more potential in-
ferences.

A **potential inference** is an inference which is not
pronounced, but which *can be* pronounced. The **formal
object** of a potential inference is a formal object
which is not asserted, but which *can be* asserted, by
an act of inference.

Examples of inference in its **primary** manifesta-
tion: "If John committed this robbery, then he is
deserving of imprisonment;" "If all anarchists are un-
patriotic, and Peter is an anarchist, then Peter is un-
patriotic."

Examples of inference in its **secondary** manifesta-
tion: "If it is true that, if it has rained, the grass
is wet, then it is true that, if the grass is not wet, it
has not rained;" "If it is true that, if John committed
this robbery, he is deserving of imprisonment, and
that, if he was in the house at midnight, he committed
this robbery, then it is true that, if John was in the
house at midnight, he is deserving of imprisonment."

From the inference, "If all anarchists are unpa-
triotic, and Peter is an anarchist, then Peter is unpa-
triotic," we can derive by eduction the following: "If

it is true that Peter is not unpatriotic, then it is true that, if all anarchists are unpatriotic, Peter is not an anarchist." In this second inference we assert the logical dependence of a potential inference upon a potential judgment.

Hereafter, for the sake of convenience, we shall refer chiefly to the act of inference in its *primary* manifestation; for what we shall say of it under this form will apply to it *mutatis mutandis* in its secondary manifestation.

The **consequent** is the formal object of a potential judgment which is asserted to be logically dependent upon the formal object of one or more potential judgments. More briefly, it is a potential judgment which is asserted to be logically dependent upon one or more potential judgments. Thus, in the inference, "If John committed this robbery, then he is deserving of imprisonment," the consequent is "he is deserving of imprisonment."

The **antecedent** is the formal object of the one or more potential judgments upon which the consequent is asserted to be logically dependent. Less accurately, it is the potential judgment or potential judgments upon which the consequent is asserted to be logically dependent. Thus, the antecedent of the foregoing inference is the potential judgment, "John committed this robbery."

The formal object of a potential judgment is **logically dependent** upon the formal object of another potential judgment, *when the reality of the latter involves the reality of the former,* and consequently, *when the assertion of the latter involves the assertion of the former.*

Instead of the cumbersome expression, "the reality of the formal object of the one or more potential judgments upon which the consequent is asserted to be logically dependent," we shall adopt as its equivalent the shorter phrase, "the truth of the antecedent." We may then construct a concise definition of logical dependence as follows:

Logical dependence or **sequence** means that the truth of the antecedent involves the truth of the consequent.

Note.—The act of inference is not a judgment; for it is not the mental assertion of the objective identity or diversity of two ideas (cf. 19). What is implied in the inference we have been employing in illustration is that the evidence which shall warrant our pronouncing John guilty of this robbery will also warrant our pronouncing him deserving of imprisonment. But in the inference itself we do not make either of these pronouncements, and hence the inference does not contain a judgment. In inference the mind prescinds from the presence or absence of any evidence for pronouncing any of the potential judgments contained in the matter of the inference. One or more of the judgments whose formal objects are contained in the matter of the inference may have been made *prior* to the inference; but in the act of inference itself none of these judgments is made. Frequently the inference is called a conditional judgment. It would be more correct to say that it contains the formal object of a judgment which is held in abeyance till the antecedent is proved; and a judgment in abeyance is not a judgment at all. There is no such thing as a conditional judgment, any more than there is a conditional act of jumping.

When it is contended that the example we have been using is a conditional judgment, it is not meant that the hypothetical proposition can be resolved into a categorical; for, when it is resolved into a categorical proposition, no one would call it the expression of a *conditional* judgment. What is meant by the contention we refer to is that in our example we *assert conditionally* that "John is deserving of imprisonment." As a matter of fact, in that example we do not *assert* at all, whether conditionally or otherwise, that "John is deserving of imprisonment;" just as in the second example we do not assert that "Peter is unpatriotic."

47. An **à priori inference** is an inference which asserts a logical dependence perceived by means of a mere comparison of the formal objects of the potential judgments which enter into it either with themselves or with the formal object of another potential judgment; e.g. "If he walked along a straight line from one point to another, then he travelled over the shortest distance between those two points;" "If a line is drawn through a circle perpendicular to a tangent from the point of junction between the tangent and the circle, then the line will pass through the centre of the circle."

An **à posteriori inference** is an inference which asserts a logical dependence perceived by means of experience in addition to comparison; e.g. "If the iron is hot, then it will melt the wax;" "If a feather and a coin are allowed to fall together in a vacuum, then they will descend at the same rate of speed."

An **immediate inference** is an inference which asserts a logical dependence perceived without recourse to

proof; e.g. "If he walked along a straight line from one point to another, then he travelled over the shortest distance between those two points."

A **mediate inference** is an inference which asserts a logical dependence perceived by means of proof; e.g. "If a line is drawn through a circle perpendicular to a tangent from the point of junction between the tangent and the circle, then the line will pass through the centre of the circle."

CHAPTER VII

THE HYPOTHETICAL PROPOSITION

48. The **hypothetical** or **conditional proposition** is the normal verbal expression of the matter and form of an act of inference; e.g. "If John committed this robbery, then he is deserving of imprisonment."

The *matter* of an inference, that is, the two or more potential judgments which enter into it, are expressed by the two parts of the proposition called **antecedent** and **consequent.** The antecedent and consequent of the hypothetical proposition correspond to the antecedent and consequent of the act of inference. In the foregoing example the antecedent is "John committed this robbery," and the consequent is "he is deserving of imprisonment." Since the antecedent and consequent express the matter of the inference, they are called the **matter** of the hypothetical proposition.

The *form* of an inference, that is, the sequence or logical dependence, is expressed by "If . . . then," though it is not unusual to omit one or both of these particles. For this reason, "If . . . then" is called the **form** of the hypothetical proposition."

The hypothetical proposition is neither affirmative nor negative; for it asserts neither the objective identity nor the objective diversity of two ideas (cf. 19). The presence or absence of a negative particle in the antecedent or the consequent or both does not affect

63

the *form* of the hypothetical proposition. "If this ani-
mal is not rational, then it is not a man:"—this prop-
osition is neither affirmative nor negative.

We do, however, speak of *denying* a hypothetical
proposition. But denying such a proposition does not
mean making it negative; it means pronouncing it
false. The act of the mind which pronounces a hypo-
thetical proposition false is sometimes a judgment and
sometimes an inference. In general, we may say that
any pair of propositions are **contradictories,** which are
such that they cannot both be true nor both be false;
e.g. "If a pupil is studious, he is successful"—"Some-
times if a pupil is studious, he is not successful;" "If
a pupil is studious, he is successful"—"Some studious
pupils are not successful." If the propositions are
such that they cannot both be true, but both may be
false, at the same time, they are **contraries;** e.g. "If a
pupil is studious, he is successful"—"If a pupil is
studious, he is not successful;" "If a pupil is studious,
he is successful"—"No studious pupil is successful."

A hypothetical proposition is **true,** when the
sequence or *logical dependence* which it asserts is real;
is **false,** when the sequence is not real (cf. 19).

A hypothetical proposition may be true, though the
formal objects of the potential judgments entering into
it are not only unreal, but impossible; e.g. "If this
circle is square, it has four right angles."

49. If the same term stands as subject in the ante-
cedent and the consequent, and is not a singular term,
the hypothetical proposition can easily be resolved
into a categorical; e.g. "If a man is just, he is brave"
—"All just men are brave." In like manner, universal

and particular categorical propositions can frequently be resolved into hypotheticals. Thus, by resolving the propositions which were used in the square of categorical opposites in section 36, we may construct a square of hypothetical opposites:

1. If a man is a soldier, he is brave.
2. If a man is a soldier, he is not brave.
3. Sometimes if a man is a soldier, he is brave.
4. Sometimes if a man is a soldier, he is not brave.

1 and 4 and also 2 and 3 are **contradictories**; 1 and **2** are **contraries**; 3 and 4 are **subcontraries**; 1 and 3 and also 2 and 4 are **subalterns**.

In this connection some remark should be made concerning propositions like the one which in the next section is symbolized thus: "If X is true, then Y is true." The opposition between propositions of this kind may be illustrated as follows:

1. If X is true, Y is true.
2. If X is true, Y is not true.
3. If X is true, Y may be true.
4. If X is true, Y need not be true.

If 3 and 4 be contemplated, not by themselves, but simply as the contradictories of 2 and 1 respectively, they may also be written as follows: "Even though X is true, still Y may be true," "Even though X is true, still Y need not be true."

50. The **import** or **meaning** of a hypothetical proposition is that which is explicitly asserted by the inference which it expresses; it is that which is adverted to in the act of inferring. In other words, it is the formal object of the inference.

The **implication** of a hypothetical proposition is an inference or several inferences involved in the import of the proposition.

If we allow A, B, C, D to stand for terms, the hypothetical proposition will usually have the form, "If A is B, C is D;" e.g. "If this child is disobedient, his parents suffer." When the antecedent and the consequent have the same subject, the form will be "If A is B, it is D;" e.g. "If this child is disobedient, he is selfish." If we allow X and Y to stand for antecedent and consequent respectively, the proposition will have the form, "If X is true, then Y is true;" e.g. "If the earth is immovable, the sun moves round the earth." This last form may be written still more briefly, thus: "If X, then Y."

We saw in section 46 that what is asserted by an act of inference is that the truth of the antecedent involves the truth of the consequent. But in this act we neither assert nor imply that the truth of the consequent involves the truth of the antecedent. Thus, in the proposition, "If this child is disobedient, his parents suffer," we neither assert nor imply that "If the parents of this child suffer, he is disobedient." The parents might suffer for many reasons without the child being disobedient.

If, however, the truth of the antecedent is a **necessary condition** of the truth of the consequent, then the truth of the consequent involves the truth of the antecedent. For example, "If this triangle is equilateral, all its angles are equal;" from this we may conclude, "If all the angles of this triangle are equal, it is equilateral." However, the presence of a necessary con-

dition cannot be learned from the *form* of the proposi-
tion, but only from an inspection of its *matter.*

N. B.—The *matter* of propositions of a given kind
varies indefinitely, but the *form* is the same in all of
them. Hence, whatever we can learn from the *form*
of any proposition we can learn from the form of every
proposition of the same kind.

From the fact that if the antecedent is true, the
consequent is true, it follows that if the consequent
is false, the antecedent is false; for, if the antecedent
were not false, it would be true, and that would make
the consequent true, which is against the supposition.
For example, from the proposition, "If this child is
disobedient, his parents suffer," we may pass to the
proposition, "If the parents of this child do not suffer,
he is not disobedient."

The falsity of the antecedent does not involve the
falsity of the consequent, except in the case of the
necessary condition. For example, "If this child is
disobedient, his parents suffer:" from this proposition
we cannot argue, "If this child is not disobedient, his
parents do not suffer."

We may sum up these results as follows: The prop-
osition, "If A is B, C is D," yields "If C is not D, A is
not B," but neither of the following propositions: "If
C is D, A is B," "If A is not B, C is not D."

THE DISJUNCTIVE PROPOSITION

51. The **disjunctive** or **alternative proposition** is the
verbal expression of the matter and form of an act of
inference, the form being expressed by the particles

"either . . . or;" consequently, it can be resolved into a hypothetical proposition. For example, "Either George or Peter is a lawyer" = "If George is not a lawyer, Peter is," "If Peter is not a lawyer, George is;" "George is either a lawyer or a physician" = "If George is not a lawyer, he is a physician," "If George is not a physician, he is a lawyer;" "Either the earth moves round the sun, or astronomy is an illusion" = "If the earth does not move round the sun, astronomy is an illusion," "If astronomy is not an illusion, the earth moves round the sun." These three examples may be represented symbolically as follows: "Either A or B is C," "A is either B or C," "Either X is true or Y is true."

The elements of the disjunctive proposition connected by "or" are called **Alternants.**

The alternants are the **matter,** and "either . . . or" is the **form,** of the disjunctive proposition.

So far as we are able to judge from the *form* of the disjunctive proposition, the alternants are not mutually exclusive. Thus, in the proposition, "The pupil is either diligent or talented," we do not mean to exclude the possibility of the pupil being both diligent and talented (cf. 50, N. B.). Sometimes, however, we see from the alternants themselves—that is, from the *matter* of the proposition—that they are mutually exclusive; e.g. "He either passed or failed in the examination." A proposition of this kind yields four hypotheticals, namely: "If he did not pass in the examination, he failed," "If he did not fail in the examination, he passed," "If he passed in the examination, he did not fail," "If he failed in the examination, he did not pass."

A disjunctive proposition with mutually exclusive alternants corresponds to a hypothetical with a necessary condition.

When a disjunctive proposition contains only two alternants, the rule for resolving it into a hypothetical is as follows: *Negative one of the alternants, make it the antecedent, and leave the other as it stands for the consequent.*

When a hypothetical proposition has a single antecedent and a single consequent, the rule for resolving it into a disjunctive is as follows: *Negative the antecedent, leave the consequent as it stands, and connect the two by "or."*

52. Propositions like "No man can be noble and base," "No man is both noble and base," are really disjunctive propositions with negative alternants, and may be expressed as follows: "Either a man is not noble or he is not base." This proposition is resolvable into the hypothetical, "If a man is noble, he is not base," "If a man is base, he is not noble."

The proposition, "John is noble," means "John possesses the attribute noble" (cf. 19). "John is non-noble" is equivalent to "John lacks the attribute noble." "No man can be noble and non-noble":—this proposition, in the ordinary disjunctive form, is written thus: "Either a man is not noble or he is not non-noble." The full interpretation of this proposition is as follows: "Either a man does not possess the attribute noble or he does not lack the attribute noble." This gives us the hypothetical, "If a man possesses the attribute noble, he does not lack the attribute noble," "If a man lacks the attribute noble, he does not possess the attribute noble."

THE FUNDAMENTAL LAWS OF THOUGHT
OR FIRST PRINCIPLES

53. In general, a **Law** is a norm which must be followed in order to reach a certain end.

A **Law of Thought** is a norm or principle with which our judgments and inferences must be in accord or at least not at variance, if we are to think correctly.

The laws of thought are countless in number, and the vast majority of them have never been formulated. When they are formulated, they are frequently self-evident. Even the laws that have been formulated we follow for the most part without explicit advertence to them. Nevertheless, if in a given process of thought we violate any one of them, the process will be chaotic and fruitless.

Three laws of thought are usually set down by logicians as being the most fundamental of all, viz. the Law of Identity, the Law of Contradiction, and the Law of Excluded Middle. The first is a judgment; the second and third, as we shall see presently, are acts of inference.

LAW OF IDENTITY: *Everything is what it is, or Everything is itself.*

This law emphasizes the unchangeable character of objective truth and its independence of thought and will (cf. 19). Hence, once a truth is ascertained, we are not at liberty to disregard it in our judgments and inferences.

LAW OF CONTRADICTION: *Nothing can at the same time and in the same respect be and not be.*

This proposition expresses an act of inference; for we saw in the preceding section that its ordinary disjunctive form is as follows: "Either a thing does not possess a certain attribute or it does not lack that attribute;" and this is resolvable into the hypothetical, "If a thing possesses a certain attribute, it does not lack that attribute," "If a thing lacks a certain attribute, it does not possess that attribute."

In the usual formulation of the law special attention should be called to the words, *"at the same time and in the same respect,"* which are inserted in order to eliminate the ambiguity which would otherwise lurk in this formula.

LAW OF EXCLUDED MIDDLE: *Everything either is or is not.*

This again is the expression of an act of inference. The meaning of the proposition, more fully expressed, is, "Either a thing possesses a certain attribute or it lacks that attribute." Casting this into strictly logical form, we have the hypothetical proposition, "If a thing does not possess a certain attribute, it lacks that attribute," "If a thing does not lack a certain attribute, it possesses that attribute."

We saw in section 49 that when a hypothetical proposition has the same general term as subject in the antecedent and the consequent, it can be resolved into a categorical. Hence, the Law of Contradiction and the Law of Excluded Middle may be stated as follows:

Law of Contradiction: "A thing which possesses a certain attribute does not lack that attribute," "A thing which lacks a certain attribute, does not possess

that attribute." More briefly: *"A thing with a certain attribute is not a thing without that attribute,"* *"A thing without a certain attribute is not a thing with that attribute."*

Law of Excluded Middle: "A thing which does not possess a certain attribute lacks that attribute," "A thing which does not lack a certain attribute possesses that attribute."

CHAPTER VIII

THE PROCESS OF INFERENCE AND THE SYLLOGISM

54. The **process of inference** is the process of establishing the reality of the formal object of a judgment or of an act of inference (cf. 19). More briefly, it is the process of establishing the truth of a judgment or of an act of inference.

The process of inference involves three acts, viz. an act of inference, the assertion of the antecedent of this act of inference, and the assertion of its consequent. For example—

> If the defendant is innocent, the court should acquit him,
>
> But the defendant is innocent,
>
> Therefore the court should acquit him.

In this example the act of inference is contained in the first line, and is called the **Major Premise.** The assertion of the antecedent is in the second line, and is called the **Minor Premise.** The assertion of the consequent is in the third line, and is called the **Conclusion.**

Since the consequent is known to be logically dependent upon the antecedent, the function of **proof** is to furnish evidence of the antecedent; for, once the antecedent is proved, the consequent also is proved (cf. 46). In many cases the proof is at hand before

the act of inference is made.   Speaking figuratively, all proof consists in the elimination of the antecedent, thus leaving us to assert the consequent.   The antecedent is eliminated in the minor premise.

The consequent or conclusion is **true,** when its formal object is a reality (cf. 19).   It is **valid,** when it is logically dependent upon the antecedent.

55. An **argument** is the verbal expression of one or more processes of inference.

The **syllogism** is an argument which expresses only one process of inference, and in this order: the act of inference, the assertion of the antecedent, and the assertion of the consequent.   Because of its conciseness and accuracy in exhibiting the reasoning process, the syllogism is called the perfect expression of the process of inference.

N. B.—The name "syllogism" is also applied to one or two forms of argument which, as they stand, are an incomplete expression of a process of inference. We shall refer to this again in sections 56 and 62 and also in the Appendix.   Cf. APPENDIX: *Note on Section 55.*

The syllogism is composed of three propositions which are called **Major Premise, Minor Premise,** and **Conclusion,** corresponding to the major premise, minor premise, and conclusion of the reasoning process.   The example which was given in the preceding section to illustrate the process of inference will also serve to illustrate the syllogism.

If the major or the minor premise or the conclusion is omitted from the verbal expression of the reasoning process, the argument is called an **Enthymeme.**

CHAPTER IX

THE MIXED HYPOTHETICAL SYLLOGISM

56. We begin the explanation of the syllogism with the mixed hypothetical syllogism, because this is the least complex of all forms of argument.

The **mixed hypothetical** or **mixed conditional syllogism** is a syllogism in which the major premise is a hypothetical proposition with a single antecedent and a single consequent, and the minor premise, a simple categorical proposition either positing the antecedent or sublating the consequent of the major premise.

To **posit** is to set down as true.

To **sublate** is to set down as false.

The character of this syllogism is shown in the following formulas:

> If A is B, C is D,
> A is B,
> Therefore C is D.

> If A is B, C is D,
> C is not D,
> Therefore A is not B.

There are two "moods" of the mixed hypothetical syllogism, viz. the **Modus Ponens** or **Constructive Syllogism** and the **Modus Tollens** or **Destructive Syllogism.** In the *modus ponens* the minor premise *posits the antecedent* of the major premise. In the

modus tollens the minor premise *sublates the consequent* of the major premise.

The *modus ponens* (or *modus ponendo ponens*) presents no difficulty. The justification of the *modus tollens* (or *modus tollendo tollens*) lies in a suppressed major premise which, if expressed, would show that the *modus tollens* is the *modus ponens*. We saw in section 50 that the proposition "If this child is disobedient, his parents suffer" yields by eduction the proposition "If his parents do not suffer, this child is not disobedient." If, then, in the *modus tollens* we express the suppressed major premise, the argument will run as follows:

> If this child is disobedient, his parents suffer,

Therefore If his parents do not suffer, this child is not disobedient,

> But his parents do not suffer,

Therefore This child is not disobedient.

This argument is now in the *modus ponens;* for the minor premise—"His parents do not suffer"—posits the antecedent of the premise immediately preceding it. The question we have just been discussing will be considered at greater length in the Appendix. (Cf. pp. 218-221).

57. Rules of the Mixed Hypothetical Syllogism:

RULE I: *Positing the antecedent in the minor premise necessitates positing the consequent in the conclusion.*

RULE II: *Sublating the antecedent in the minor premise does not warrant sublating the consequent in the conclusion.*

The reason for Rule I is that in a hypothetical proposition we assert that the consequent is logically dependent upon the antecedent, that the truth of the antecedent involves the truth of the consequent (cf. 46).

The reason for Rule II is that we do not assert that the truth of the antecedent is a *necessary condition* of the truth of the consequent.   For all we know from the *form* of the proposition, the consequent may be true, though the antecedent is false (cf. 50). Hence, we cannot say that the consequent is false because the antecedent is false.

RULE III: *Sublating the consequent in the minor premise necessitates sublating the antecedent in the conclusion.*

RULE IV: *Positing the consequent in the minor premise does not warrant positing the antecedent in the conclusion.*

The reason for Rule III is that the *modus tollens* becomes the *modus ponens* when the suppressed major premise is expressed; and sublating the original consequent is the same as positing the antecedent of the suppressed major premise.

The reason for Rule IV is that in a hypothetical proposition we do not assert that the truth of the antecedent is a *necessary condition* of the truth of the consequent, and hence, so far as we know from the *form* of the proposition, the consequent may be true without the antecedent being true.

If in a given hypothetical proposition the *matter* reveals that the truth of the antecedent is a *necessary condition* of the truth of the consequent, then we are justified in positing the consequent in the minor

premise and the antecedent in the conclusion; we may also in this case sublate the antecedent in the minor premise and the consequent in the conclusion.

The first and third rules are illustrated in the following examples:

Modus ponens: If it has rained, the grass is wet,
　　　　　　　　　But it has rained,
　　Therefore The grass is wet.

<p align="center">or</p>

Modus tollens: But the grass is not wet,
　　Therefore It has not rained.

Modus ponens: If the boy has not studied, he will fail,
　　　　　　　　　But the boy has not studied,
　　Therefore He will fail.

<p align="center">or</p>

Modus tollens: But the boy will not fail,
　　Therefore He has studied.

Modus ponens: If the general is skilful, he will not lose the battle,
　　　　　　　　　But the general is skilful.
　　Therefore He will not lose the battle.

<p align="center">or</p>

Modus tollens: But the general will lose the battle,
　　Therefore He is not skilful.

Caution: When the consequent is sublated in the minor premise, the conclusion will be the *contradictory*, not the *contrary*, of the antecedent. Sublating the consequent in the minor premise warrants us only in asserting that the antecedent is *not true*, that is, in

asserting that its *contradictory* is true.

> If all men were upright, crimes would not
>   occur,
> But crimes do occur,

Therefore Some men are not upright.

It would be a fallacy to conclude, "Therefore no men are upright."

58. **Fallacies of the Mixed Hypothetical Syllogism.** There are two fallacies which may easily be committed by the unwary in using the mixed hypothetical syllogism:

1. The fallacy of **sublating the antecedent** in the minor premise, that is, of deducing the falsity of the consequent from the falsity of the antecedent. For example—

> If it has rained, the grass is wet,
> But it has not rained.

Therefore The grass is not wet.

2. The fallacy of **positing the consequent** in the minor premise, that is, of deducing the truth of the antecedent from the truth of the consequent. For example—

> If it has rained, the grass is wet,
> But the grass is wet,

Therefore It has rained.

THE DISJUNCTIVE SYLLOGISM

59. The **disjunctive** or **alternative syllogism** is a syllogism in which the major premise is a disjunctive

proposition, and the minor premise, a proposition sublating part of the alternants of the major premise. For example—

> A is either B or C,
> A is not B,
> Therefore A is C.

> A is either B or C or D,
> A is not B,
> Therefore A is either C or D.

> A is either B or C or D,
> A is neither B nor C,
> Therefore A is D.

> Either X is true or Y is true,
> Y is not true,
> Therefore X is true.

The minor premise in each of the foregoing syllogisms *sublates* part of the alternants of the disjunctive premise, and the conclusion *posits* what remains. A syllogism of this type is called the **Modus Tollendo Ponens.**

As was pointed out in section 51, the *form* of a disjunctive proposition does not warrant us in interpreting the alternants as mutually exclusive. In the proposition, "He was either first or second in the race," the alternants are mutually exclusive; but this is revealed by the *matter* of the proposition, not by its *form*.

In the **Modus Ponendo Tollens** the minor premise *posits* part of the disjunctive premise, and the conclusion *sublates* the remainder.

The *modus tollendo ponens* is always valid.

The *modus ponendo tollens* is valid only when the alternants are *known* to be mutually exclusive.

Modus tollendo ponens—Valid:

> He is either diligent or talented,
> He is not talented,

Therefore He is diligent.

Modus ponendo tollens—Invalid:

> He is either diligent or talented,
> He is talented,

Therefore He is not diligent.

In the first paragraph of section 51 it was observed that the disjunctive proposition is resolvable into the hypothetical. Hence, the disjunctive syllogism with but two alternants is only the mixed hypothetical syllogism worded differently. Expressing the foregoing examples in hypothetical form, we shall find that the *modus tollendo ponens* is either the *modus ponens* or the *modus tollens* of the mixed hypothetical syllogism, and that the *modus ponendo tollens* commits one of the fallacies indicated in the preceding section.

The *modus tollendo ponens* becomes:

> If he is not talented, he is diligent,
> He is not talented,

Therefore He is diligent.

The *modus ponendo tollens* becomes:

> If he is not talented, he is diligent,
> He is talented,

Therefore He is not diligent.

N. B.—*Contradictory* alternants are not only mutually exclusive, but collectively exhaustive (cf. 9, 36, 37);

e.g. "Either it rains or it does not rain;" "He is either learned or not learned."

60. The Conjunctive Syllogism. The so-called **conjunctive syllogism** is a disjunctive syllogism in which the alternants of the disjunctive premise are all negative. For example—

> Every gem is either not a diamond or not a
> ruby,
> This gem is a diamond,

Therefore It is not a ruby.

The major premise is usually stated as in the following syllogism, and it is when this premise is so stated that the syllogism is called conjunctive:

> No gem can be both a diamond and a ruby,
> This gem is a diamond,

Therefore It is not a ruby.

CHAPTER X

THE SIMPLE CATEGORICAL SYLLOGISM

61. The **simple categorical syllogism** is the verbal expression of a process of inference that contains an act of inference in which is asserted the real logical dependence of the objective identity or diversity of two ideas upon two potential judgments in which the formal objects of the two ideas are compared separately with the formal object of a third idea.

The simple categorical syllogism, as commonly set forth in text-books on Logic, expresses merely three judgments, the sequence or logical dependence being indicated by the particle "therefore." In the two first judgments the formal objects of two ideas are compared separately with the formal object of a third idea and their identity with or diversity from the formal object of the third idea is separately asserted (cf. 19, N. B.). In the third judgment the objective identity or diversity of the two first ideas is asserted. The propositions expressing the two first judgments are called the **antecedent** or **premises** of the syllogism. The proposition expressing the third judgment is called the **consequent** or **conclusion**.

This type of syllogism is called *categorical,* because the three propositions which usually compose it are

categorical.   For example—

> All men are mortal,
> But all kings are men,
> Therefore All kings are mortal.

62.   It is important to observe that, if we express
in full the process of inference which issues in the
conclusion, "All kings are mortal," then the two prop-
ositions, "All men are mortal" and "All kings are
men," will form one compound categorical proposition
in the minor premise, and the major premise will be
a hypothetical proposition expressing an act of in-
ference in which the antecedent consists of two poten-
tial judgments comparing the formal objects of two
ideas separately with the formal object of a third idea,
and the consequent consists of a potential judgment
whose formal object is the objective identity or diver-
sity of those two ideas (cf. APPENDIX: *Note on Sec-
tion 55*).   Thus:

> If all men are mortal and all kings are men,
>   then all kings are mortal,
> But all men are mortal and all kings are
>   men,
Therefore All kings are mortal.

In text-books on Logic it is customary to omit the
major premise of the foregoing syllogism and to treat
the two simple propositions constituting the com-
pound proposition of the minor premise as two sep-
arate premises.   It will be convenient to follow this
practice, and we shall write the simple categorical
syllogism as it was written in the preceding section.

63. The following are the two axioms on which the simple categorical syllogism is based:

AXIOM OF IDENTITY: *When the formal objects of two ideas are identical with the formal object of a third idea, they are identical with each other.*

AXIOM OF DIVERSITY: *When the formal objects of two ideas are one of them identical with and the other different (distinct) from the formal object of a third idea, they are different from each other.*

It is of the essence of the simple categorical syllogism that the formal objects of two ideas be compared with the formal object of a third idea. The formal object of the third idea forms the common basis for comparing the formal objects of the two other ideas with each other in such a way as to determine their identity or diversity. The whole purpose of the treatment of the simple categorical syllogism is to find out in what way the formal objects of two ideas may be related as regards identity or diversity with the formal object of a third idea so that their own identity or diversity shall be a *necessary sequence* from that relation (cf. 19, N. B.).

If, instead of comparing the formal objects of two ideas with the formal object of a third idea, we compared one of them with the formal object of a third, and the other with the formal object of a fourth idea, the comparison would be useless, and we should be unable to determine anything as regards the objective identity or diversity of the two first ideas; for in that case we should not be employing a basis of comparison which was *common* to the formal objects of those two ideas.

MATTER AND FORM OF THE SIMPLE CATE-
GORICAL SYLLOGISM

64. The **form** of the categorical syllogism is the *sequence,* that is, the logical dependence of the conclusion or consequent upon the premises or antecedent. If the sequence is lacking, we have only an *apparent* syllogism, not a real one.

The **proximate matter** of the simple categorical syllogism are the three propositions which enter into it.

The **remote matter** are the terms which constitute the matter of the three propositions. The terms are three, two of which are compared separately with the third term. The third term is called the **Middle Term.** The two other terms are called the **Extremes.**

When we speak of the *matter* of the syllogism without qualification, we mean the *proximate matter*.

The syllogism we have already employed may be represented symbolically as follows:

All men are mortal,   All M is P
But all kings are men,   All S is M
Therefore All kings are mortal.   Therefore All S is P

There are three propositions. These propositions contain three terms, each appearing *twice*: the *middle term*, twice in the two first propositions, but not in the third proposition; the *extremes,* each once in the third proposition and once in one of the other propositions.

The **premises** or **antecedent** are the propositions in which the extremes are compared with the middle term.

The **conclusion** or **consequent** is the proposition

which asserts the objective identity or diversity of the ideas expressed by the extremes.

The **major term** or **major extreme** is the *predicate* of the conclusion. It is denoted by the letter P.

The **minor term** or **minor extreme** is the *subject* of the conclusion. It is denoted by the letter S.

The predicate of the conclusion is called the *major* term, because the predicate of an affirmative proposition generally has a wider extension than the subject (cf. 27).

The **major premise** is the premise which contains the major term (P).

The **minor premise** is the premise which contains the minor term (S).

The order in which the premises are arranged does not affect the validity of the syllogism. Sometimes the minor premise is placed first; but usually it is found after the major premise.

LAWS OF THE TRUTH AND FALSITY OF VALID CONCLUSIONS

65. The conclusion is **valid,** when it is logically dependent upon the premises.

The conclusion is **true,** when what it asserts is a reality (cf. 19).

Special attention should be directed to the two following laws, which relate to the truth and falsity of valid conclusions:

I. *A valid conclusion from true premises is always true.*

This law follows from the very nature of a sequence

or logical dependence (cf. 46, 50), which means that the truth of the antecedent (or premises) involves the truth of the consequent (or conclusion). Hence, *if a valid conclusion is false, at least one of the premises is false.*

II. *A valid conclusion from premises one or both of which are false may be false and may be true.*

This law follows from what was said in section 50, namely, that unless the truth of the antecedent is a *necessary condition* of the truth of the consequent, the falsity of the antecedent does not involve the falsity of the consequent (or conclusion), nor does the truth of the consequent involve the truth of the antecedent.

The following examples illustrate the law:

"Every science is useless; But physics is a science; Therefore physics is useless." Here a *valid but false* conclusion is derived from premises one of which is false.

"No horses are able to fly; But all dogs are horses; Therefore no dogs are able to fly." Here we have a conclusion which is *both valid and true* derived from premises one of which is false.

"Every stone is rational; But every man is a stone; Therefore every man is rational." In this syllogism both premises are false, but the conclusion is *both valid and true.*

Two important practical corollaries flow from the second law:

(1)   *It does not follow that a doctrine is false because the arguments adduced in support of it are false.*

(2)   *It does not follow that the arguments adduced*

in support of a doctrine are true because the doctrine itself is true.

A conclusion is true **per se,** when it is not only valid and true, but follows from true premises.

A conclusion is true **per accidens,** when it is true, but not valid, and also when it is both true and valid, but follows from premises one or both of which are false.

CHAPTER XI

FIGURES AND MOODS OF THE CATEGORICAL SYLLOGISM

66. The **figure** of a categorical syllogism is the relation between the position of the middle term in the major premise and its position in the minor.

If we state the major premise first, and denote the major, middle, and minor terms by the letters P, M, S respectively, we may arrange the terms of the premises in four different ways so as to obtain four figures, as follows:

Fig. 1	Fig. 2	Fig. 3	Fig. 4
M —— P	P —— M	M —— P	P —— M
S —— M	S —— M	M —— S	M —— S
S —— P	S —— P	S —— P	S —— P

In the **first figure** the middle term is subject in the major premise and predicate in the minor.

In the **second figure** the middle term is predicate in both premises.

In the **third figure** the middle term is subject in both premises.

In the **fourth figure** the middle term is predicate in the major premise and subject in the minor.

The diagram below shows the relative positions of the middle term in the four figures, the letters X and Y

denoting respectively the subject and the predicate of
the major premise, and X' and Y', the subject and the
predicate of the minor.
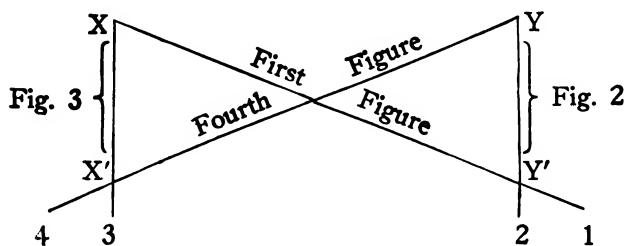
If the lines are extended through X' and Y' and
their extremities numbered backwards, the lines num-
bered 1, 2, 3, 4 indicate the relative positions of the
middle term in the first, second, third, and fourth fig-
ures respectively.

The four figures are illustrated concretely in the
following examples:

Figure 1:   All men are mortal,
            All kings are men,
Therefore All kings are mortal.

Figure 2:   All animals are sentient,
            No plants are sentient,
Therefore No plants are animals.

Figure 3:   All men are mortal,
            All men are rational beings,
Therefore Some rational beings are mortal.

Figure 4:   No soldiers are women,
            Some women are brave,
Therefore Some brave persons are not soldiers.

67. The **mood** of a categorical syllogism is the logical dependence of the quality and quantity of the conclusion upon the quality and quantity of the premises. For example, AII is a mood in which the major premise is universal affirmative, the minor premise, particular affirmative, and the conclusion, particular affirmative.

It now remains for us to determine the rules of the different figures and to find out what combinations of quality and quantity in the premises of each figure will yield a valid conclusion. In undertaking to do this, we suppose that the following conditions are fulfilled: (1) that the predicate of the major premise is not a singular term; (2) that "some" in the sense of "one at least" is the only sign of a particular proposition employed in the syllogism; (3) that we have no information about the *matter* of a proposition, but only about its quality and quantity. It is necessary to presuppose these conditions in order to simplify the explanation of the categorical syllogism.

DICTA OF THE THREE FIRST FIGURES

68. The usual method of procedure in works on Logic is first to prove the general rules of the syllogism and then to take up the figures and moods. But the whole treatment of the categorical syllogism will be much simpler and more intelligible if this method is reversed. In the method which is commonly adopted the explanation of the syllogism is needlessly complicated. Moreover, in this method the proof of the general rules proceeds on the hypothesis that the predicate is undistributed in affirmative,

and distributed in negative, propositions (cf. APPEN-
DIX: *Note on Section 29*).   Again, the general rules, as
usually derived, do not of themselves make it evident
that a syllogism which conforms to them is valid: all
they do is to warn us that a syllogism cannot violate
them without being invalid.

If, instead of following the usual order, we start
with the *dicta* of the three first figures, we shall find
that our treatment of the categorical syllogism is
clear of these disadvantages.   In the first place, by
means of the *dicta* we make evident from the outset
the validity of arguing in any of the three first figures.
Secondly, we get rid of any need of referring to the
predicate as distributed or undistributed.   Thirdly,
the rules of the different figures will be evident from
their respective *dicta*.   Fourthly, the *dicta* will supply
us immediately with all the moods of the three first
figures.   Fifthly, the general rules of the syllogism will
be seen to follow at once as corollaries from the *dicta*.

69. Aristotle laid down the **Dictum de omni et nullo**
as the axiom upon which rests the validity of the
categorical syllogism.   But it applies only to the first
figure.   This *dictum* has been worded in various ways
by different authors.   Our own wording of it will be
found in the *dictum* of the first figure.

For the sake of clearness, the letters P, M, S will
be inserted in brackets in the statement of the *dicta*.
It must be borne in mind that P denotes the *predicate*
of the conclusion, and that the premise in which it is
compared with M is the *major* premise.   Again, S
denotes the *subject* of the conclusion and is compared
with M in the *minor* premise.

In section 19 we saw that judgment may be defined as an act of the mind asserting that the object represented by one idea possesses or lacks the attribute represented by another idea. For example, "Man is rational" = "Man *possesses* the attribute rational;" "The horse is not rational" = "The horse *lacks* the attribute rational." When the words "possess" and "lack" occur in the *dicta*, they signify affirmative and negative propositions respectively.

Dictum of the First Figure: "*Any attribute* [P] *which is affirmed or denied of the formal object* [M] *of a universal idea may be affirmed or denied respectively of anything* [S] *which possesses that formal object* [M]." Cf. 23, N. B.

(More exactly stated, the *dictum* would run as follows: "Any object with a given attribute [P] which is asserted to be identical with or different from the formal object [M] of a universal idea may be asserted respectively to be identical with or different from anything [S] which is identical with that formal object [M]").

"Man" is the formal object of a universal idea. "This man" is the formal object of a singular idea. The formal object of an idea is that which is explicitly represented by the idea (cf. 2). A universal idea is an idea which represents severally many individuals, and hence it can be predicated of each of them (cf. 8).

We may illustrate the *dictum* of the first figure by a concrete example:

Every man is mortal,
Every king is a man,
Therefore Every king is mortal.

If we insert in the *dictum* the terms of this syllogism, the *dictum* will run as follows: "Any attribute ['mortal'] which is affirmed of the formal object ['man'] of a universal idea may be affirmed of anything ['every king'] which possesses that formal object ['man']."

Dicta of the Second Figure: 1. *"The formal object [P] of a universal idea which possesses a certain attribute [M] may be denied of anything [S] which lacks that attribute [M]."*

2. *"The formal object [P] of a universal idea which lacks a certain attribute [M] may be denied of anything [S] which possesses that attribute [M]."*

(The two *dicta* of the second figure may be combined into one as follows: "The formal object [P] of a universal idea which possesses or lacks a certain attribute [M] may be denied of anything [S] which respectively lacks or possesses that attribute [M]").

The second *dictum* of the second figure is illustrated in the following syllogism:

> No plant is sentient,
> Every animal is sentient,
> Therefore No animal is a plant.

"The formal object ['plant'] of a universal idea which lacks a certain attribute ['sentient'] may be denied of anything ['every animal'] which possesses that attribute ['sentient']."

Dicta of the Third Figure: 1. *"Any attribute [P] which is affirmed or denied of the formal object [M] of a universal idea which [M] in any case possesses a second attribute [S] may be affirmed or denied respectively of something [S] having the second attribute."*

2. *"Any attribute [P] which in some cases is affirmed
or denied of the formal object [M] of a universal idea
which [M] always possesses a second attribute [S] may
be affirmed or denied respectively of something [S] hav-
ing the second attribute."*

The following syllogism will serve to illustrate the
first *dictum* of the third figure:

> Every man is mortal,
> Every man is rational,
> Therefore Some rational beings are mortal.

"Any attribute ['mortal'] which is affirmed of the
formal object ['man'] of a universal idea which ['man']
in any case possesses a second attribute ['rational']
may be affirmed of something ['some rational beings']
having the second attribute."

70. The *dicta* of the first and second figures are self-
evident.   The *dicta* of the third figure are at least as
evident as the converse of an affirmative proposition.

The *dicta* of the second figure amount to this: "An
object without a certain attribute is not an object with
that attribute," "An object with a certain attribute is
not an object without that attribute."   Thus, it will
be observed that the *dicta* of the second figure are
nothing more or less than the Law of Contradiction
stated categorically (cf. 53 ad fin.).

The *dicta* of the third figure may be interpreted as
follows: 1. If "All M is S" or "Some M is S" *(minor)*,
then (by conversion) some objects with the attribute
S have the attribute M, and consequently, some ob-
jects with the attribute S have any given attribute
[P] which always accompanies M *(major =* "All M

is P") or lack any given attribute [P] which never
accompanies M *(major=*"No M is P"). 2. If "All
M is S" *(minor)*, that is, if all objects with the at-
tribute M have the attribute S, then (by conversion)
some objects with the attribute S have any given at-
tribute [P] which is sometimes combined with M
(major = "Some M is P") or lack any given at-
tribute [P] which is sometimes not combined with M
(major = "Some M is not P").

. It is to be observed that in the third and fourth
figures we must convert the minor premise in order
that S, its predicate, may become subject of the con-
clusion; but when we do convert it, we still retain in
mind all the information that was provided by the
minor premise. Hence, a particular negative proposi-
tion cannot appear as minor premise in the third or
the fourth figure (cf. 31).

It will be noticed that in the minor premise of the
last syllogism in the preceding section the term
"rational" is in the predicate position and is used abso-
lutely; in the conclusion it is in the subject position
and is used distributively (cf. 18, 8). When the sub-
ject-term of a proposition is used distributively and
is universal, e.g. "All men are mortal," we can easily
change it into the absolute use without converting
the proposition, thus, "Man is mortal;" and we can
just as easily change it back again to the distributive
use. But we cannot change the absolute use of a
predicate-term to the distributive use, unless we con-
vert the proposition.

The first and second figures involve each only one
process of reasoning. The third and fourth figures

involve two processes; for they each involve a process of conversion.   Hence, the third and fourth figures are much inferior to the first and second in point of simplicity.

The first figure was regarded by Aristotle as the most perfect; and it is so for three reasons.   First, it is the only figure in which the conclusion can be universal affirmative.   Secondly, the subject and the predicate of the conclusion occupy respectively a subject and a predicate position in the premises. Thirdly, no term is used distributively or absolutely in the conclusion which was not used in the same way in the premises (cf. 18, 8).

RULES AND MOODS OF THE FIRST FIGURE

71. In the *dictum* of the first figure we read, "Any attribute [P] which is affirmed or denied of the formal object [M] of a universal idea."   Since the words "the formal object [M] of a universal idea" are unqualified (i. e. unrestricted or unlimited), this *dictum* provides that the *major* premise shall be universal (cf. 23, N. B.).   It further provides that the *minor* premise shall be affirmative; for it says "anything [S] which *possesses* that formal object [M]."   Hence, the two rules of the first figure are—

RULE I: *The major premise must be universal.*
RULE II: *The minor premise must be affirmative.*

The *dictum* of the first figure warrants four and only four *moods*.   The words "Any attribute [P] which is

affirmed or *denied"* allow the major premise to be
either affirmative or negative; and the words *"any-*
thing [S] which possesses" allow the minor premise
to be either universal or particular affirmative. The
words "affirmed or denied *respectively"* indicate that
the conclusion is to be of the same quality as the
major premise, while the word "anything" shows that
the conclusion is to be of the same quantity as the
minor. Thus, in the first figure we have the four
following moods: AAA, EAE, AII, EIO. These
moods are contained in the following mnemonic line:

Barbara, Celarent, Darii, Ferio*que prioris* (cf. 91).

RULES AND MOODS OF THE SECOND
FIGURE

72. In the first *dictum* of the second figure we read,
"The formal object [P] of a universal idea which pos-
sesses a certain attribute [M];" and in the second
dictum, "The formal object [P] of a universal idea
which lacks a certain attribute [M]." Since the words
"The formal object [P] of a universal idea" are un-
restricted in both *dicta*, these *dicta* provide that the
major premise shall be universal. They further pro-
vide that one premise shall be *negative;* for in the
first *dictum* we read "anything [S] which *lacks* that
attribute [M]," and in the second *dictum,* "The formal
object [P] . . . which *lacks* a certain attribute
[M]." Again, they provide that one premise shall be
affirmative; for in the first *dictum* are the words "The
formal object [P] . . . which *possesses* a certain
attribute [M]," and in the second *dictum,* "anything

[S] which *possesses* that attribute [M]." Hence, the three rules of the second figure are—

RULE I: *The major premise must be universal.*
RULE II: *One premise must be negative.*
RULE III: *One premise must be affirmative.*

The first *dictum* of the second figure warrants two and only two *moods*. It allows a universal affirmative major premise ("The formal object [P] of a universal idea which *possesses"*) with a universal or particular negative minor *("anything* [S] which *lacks").* This *dictum*, then, gives us the moods, AEE, AOO.

In like manner, the second *dictum* warrants two and only two moods. It allows a universal negative major premise ("The formal object [P] of a universal idea which *lacks"*) with a universal or particular affirmative minor *("anything* [S] which *possesses"*). The two moods, then, warranted by the second *dictum* are EAE, EIO. These two moods together with the two allowed by the first *dictum* are contained in the following mnemonic line:

Cesare, Camestres, Festino, Baroco, *secundæ.*

RULES AND MOODS OF THE THIRD FIGURE

73. In the first *dictum* of the third figure we read, "Any attribute [P] which is affirmed or denied of the formal object [M] of a universal idea which [M] in any case possesses a second attribute [S];" and in the second *dictum*, "Any attribute [P] which in some cases is affirmed or denied of the formal object [M]

of a universal idea which [M] always possesses a second attribute [S]." Both *dicta* provide that one premise shall be *universal;* for in the first *dictum* the words "The formal object [M] of a universal idea" are unrestricted; and in the second *dictum* we read "which [M] *always* possesses a second attribute [S]." Again, both *dicta* provide that the *minor* premise shall be affirmative; for in both we read, "which [M] . . . *possesses* a second attribute [S]." Moreover, they both provide that the *conclusion* shall be particular; for both contain the words, "may be affirmed or denied of *something* [S]." Hence, the three rules of the third figure are—

RULE I: *One premise must be universal.*

RULE II: *The minor premise must be affirmative.*

RULE III: *The conclusion must be particular.*

The first *dictum* of the third figure warrants four and only four *moods.* It allows the major premise to be either universal affirmative or universal negative: *universal,* because the words "the formal object [M] of a universal idea" are unrestricted; either *affirmative* or *negative,* because the *dictum* reads "Any attribute [P] which is *affirmed* or *denied.*" Whether the major premise be affirmative or negative, this *dictum* allows the minor premise to be either universal or particular affirmative ("which [M] in *any* case possesses a second attribute [S]"). Thus, this *dictum* warrants the moods, AAI, AII, EAO, EIO.

The second *dictum* warrants two and only two moods. The major premise may be either particular affirmative or particular negative ("Any attribute [P] which in *some* cases is *affirmed* or *denied*"); but the

minor premise must be universal affirmative ("which [M] *always* possesses a second attribute [S]"). This gives us the moods IAI, OAO. The six moods which are warranted by the two *dicta* of the third figure are usually combined as follows:

Tertia, Darapti, Disamis, Datisi, Felapton, Bocardo, Ferison, *habet.*

Note.—If it be desired to prove the rules of the three first figures independently of the *dicta,* it may be done as follows:

First, we must show that two negative premises cannot yield a conclusion. When both premises are negative, the subject-object in both premises is really distinct from the predicate-object (cf. 19, 4, 22 ad fin.). Suppose the subject-object to be the same in both premises (Figure 3). Then we have no means of comparing the predicate-object of the major premise with the predicate-object of the minor; for both premises may be true, whether the predicate-objects are one and the same thing or different things. For example, the two premises "No M is P" and "No M is S" may be true, whether S is or is not P. And the same applies *mutatis mutandis* to the case in which the predicate-object is the same in both premises (Figure 2), and also to the case in which the subject-object in one premise is the same as the predicate-object in the other (Figures 1 and 4). Therefore, from two negative premises no conclusion can be drawn.

Figure 1: If M has the attribute P, it does not follow that M alone has this attribute; and hence, there

may be objects which have the attribute P and *lack* the attribute M. Consequently, if S lacks the attribute M, we cannot tell whether S lacks the attribute P; for S may be among the objects which have the attribute P and lack the attribute M. Therefore, the minor premise of the first figure cannot be negative.

When some M has the attribute P, some other M may *lack* the attribute P. Hence, if S has the attribute M, we cannot conclude that it has the attribute P; for S may be among the objects which have the attribute M and lack the attribute P. When some M lacks the attribute P, some other M may *have* the attribute P. Hence, if S has the attribute M, we cannot conclude it lacks the attribute P; for S may be among the objects which have both the attributes M and P. Therefore, the major premise of the first figure cannot be particular.

Figure 2: If both premises are affirmative, then both P and S have the attribute M. But there may be some objects which have the attribute M and *lack* the attribute P. Hence, from the fact that S has the attribute M we cannot conclude that it has the attribute P; for S may be among the objects which have the attribute M and lack the attribute P. Therefore, no conclusion can be drawn in the second figure from two affirmative premises.

When some P is not M, that is, when some P lacks the attribute M, there may be some objects *with* the attribute P which *have* the attribute M. Hence, if S has the attribute M, it does not follow that it lacks the attribute P; for S may be among the objects which have both the attributes M and P. When some P is M,

that is, when some P has the attribute M, there may be some objects *with* the attribute P which *lack* the attribute M. Consequently, if S lacks the attribute M, it does not follow that it lacks the attribute P; for S may be among the objects which have the attribute P and lack the attribute M. Therefore, no conclusion can be drawn in the second figure when the major premise is particular.

Figure 3: If M has the attribute P, it does not follow that M alone has this attribute; and hence, there may be objects which *have* the attribute P and *lack* the attribute M. Consequently, if no M is S, that is, if every M lacks the attribute S, which is equivalent to saying, if every S lacks the attribute M, we cannot tell whether S lacks also the attribute P; for S may be among the objects which have the attribute P and lack the attribute M. If some M is not S, there can be no conclusion, because a particular negative cannot be converted (cf. 70). Therefore, the minor premise of the third figure cannot be negative.

If some M possesses or lacks the attribute P, and some M has the attribute S, we have no means of comparing S with P; for the M's which possess or lack the attribute P may be entirely different objects from the M's which have the attribute S. Therefore, one of the premises in the third figure must be universal.

Since the minor premise of the third figure must be affirmative, and since this premise must be converted in order to obtain S, its predicate, for the subject of the conclusion, the subject of the conclusion must receive the particular sign of quantity (cf. 38). There-

fore, the conclusion of the third figure must be particular.

RULES AND MOODS OF THE FOURTH FIGURE

74. With the possible exception of the moods EAO and EIO, the fourth figure is worthless. The position of the subject and the predicate in the conclusion is the reverse of what it was in the premises. Hence, whether a term be used distributively or absolutely in the premises, it is used in the opposite way in the conclusion (cf. 18, 8). In order, then, to frame an argument on the plan of the fourth figure, we have to twist and torture it into an unnatural shape. It may safely be said that the arguments of everyday life, whether scientific or concerned with practical questions, never assume the form of the fourth figure. We should, therefore, have contented ourselves with a bare mention of this figure, were it not the tradition to give it a place in works on Logic. As it is, it would be a waste of time to construct *dicta* for this figure; for its moods are easily derived from the moods of the first and third figures. Cf. APPENDIX: *Note on Section 74.*

The rules of the fourth figure are as vague as the general rules of the syllogism, and afford us no direct aid for the construction of syllogisms or the detection of fallacies. The following are the rules of this figure:

RULE I: *If the major premise is affirmative, the minor must be universal.*

RULE II: *If either premise is negative, the major must be universal.*

RULE III: *If the minor premise is affirmative, the conclusion must be particular.*

If we let the major and minor premises of the three first moods in the first figure become minor and major premises respectively, we shall obtain three moods in the fourth figure. Thus, by transposing the premises of *Barbara, Celarent,* and *Darii,* we obtain the following moods in figure four: AAI, AEE, IAI. Again, if we convert *simply* the major premise of the moods *Felapton* and *Ferison* in the third figure, we obtain two moods in the fourth figure, viz. EAO and EIO. These five moods are contained in the line—

Bramantip, Camenes, Dimaris, Fesapo, Fresison.

75. The moods we have mentioned are the only ones in the four figures, unless we wish to draw a particular conclusion when we may draw a conclusion which is universal. Thus, in the first figure, instead of the moods AAA and EAE, we may have AAI and EAO; in the second figure, instead of EAE and AEE, we may have EAO and AEO; in the fourth figure, instead of AEE, we may have AEO.

When we draw a particular conclusion from premises which warrant a universal conclusion, the particular conclusion is called a "weakened" conclusion, and the mood is called a **Subaltern Mood.** As we saw in the preceding paragraph, there are five subaltern moods in the four figures. Apart from the subaltern moods, the four figures have nineteen moods, which **are** enumerated in the following mnemonic lines:

Barbara, Celarent, Darii, Ferio*que prioris;*
Cesare, Camestres, Festino, Baroco, *secundæ;*
Tertia, Darapti, Disamis, Datisi, Felapton,
Bocardo, Ferison, *habet; Quarta insuper addit*
Bramantip, Camenes, Dimaris, Fesapo, Fresison.

CHAPTER XII

GENERAL RULES OF THE CATEGORICAL SYLLOGISM

76. The rules of the three first figures are direct and easy of application. Given the figure, two rules, or at the most three, are a sufficient test of the validity of a syllogism. The general rules of the syllogism, on the contrary, applying as they do to all the figures, are more indefinite in character; and, by reason of the numerous details which they require to be attended to, their employment for the detection of fallacy is much more difficult. It is plain, then, that the validity of a categorical syllogism should ordinarily be tested by the rules of the figures rather than by the general rules.

It has been observed that all the rules and moods of the three first figures are contained in their *dicta*. It will now be shown that in these *dicta* are contained also the general rules of the syllogism, so far as they are true and useful. Consequently, to know the *dicta* and their more obvious implications is to have mastered the categorical syllogism.

Since the fourth figure is unnatural, and since its moods are easily derivable from the moods of the first and third figures, its special requirements will be disregarded in the comments we are about to make upon the general rules.

108

77. The Latin wording of the general rules is as follows:

1. Terminus esto triplex: major, mediusque, minorque.
2. Latius hunc, quam præmissæ, conclusio non vult.
   (Nequaquam medium capiat conclusio oportet).
3. Aut semel aut iterum medius generaliter esto.
4. Utraque si præmissa neget, nil inde sequetur.
5. Ambæ affirmantes nequeunt generare negantem.
6, 7. Pejorem sequitur semper conclusio partem.
8. Nil sequitur geminis ex particularibus umquam.

   In English the rules run as follows:

1. The simple categorical syllogism contains three and only three terms, and is composed of three and only three propositions.
2. No term may be distributed in the conclusion which was not distributed in the premises.
   (The conclusion must not contain the middle term).
3. The middle term must be distributed at least once in the premises.
4. From two negative premises no conclusion can be drawn.
5. Two affirmative premises cannot yield a negative conclusion.
6. A negative premise requires a negative conclusion.
7. A particular premise requires a particular conclusion.
8. From two particular premises no conclusion can be drawn.

78. Few occasions occur for appealing to the rule inclosed in the parenthesis. In a categorical syllogism the conclusion merely expresses the objective identity or diversity of the two ideas which in the premises were

compared with the idea expressed by the middle term. Consequently, an argument which should express the middle term in the conclusion would not be a categorical syllogism even in appearance (cf. 45, *Note*). It should, however, be remarked in passing that the introduction of 'the middle term into the conclusion may be attended by a fallacy not unlike the one that was noticed in connection with Eduction by an Added Determinant (cf. 42). For example, "These men are tennis-players; These men are poor; Therefore these men are poor tennis-players."

Rule 3 is necessary only for the first figure. It is covered by Rule 8 for the third figure. It is meaningless when applied to the second figure. In the second figure the middle term is neither distributed nor undistributed; for, being predicate in both premises, it is not used distributively at all. Cf. APPENDIX: *Note on Section 29.*

The rule which is numbered 7 in the English wording of the Rules is superfluous. It is covered by Rule 2 for all three figures. Since the only term which can be distributed or undistributed in the conclusion is the minor term, Rule 2 means that the minor term must not be distributed in the conclusion, if it was not distributed in the premises; it can be neither distributed nor undistributed in the premises of the third figure; for in this figure it is predicate in its premise.

EXPLANATION OF THE GENERAL RULES

79. RULE I: *The simple categorical syllogism contains three and only three terms, and is composed of three and only three propositions.*

Rule 1 is provided for in all five *dicta;* for everyone of them requires three and only three terms, viz. P, M, and S, and prescribes that P and S shall be combined separately with M—thus giving us the two ·premises,—and then combined with each other—thus giving us the conclusion.

This rule follows from the very nature of the simple categorical syllogism.   Since this type of syllogism deals with the formal objects of three and only three ideas, it is evident that three and only three ideas can be expressed by the terms of any argument which is a simple categorical syllogism.

We saw in section 12 that a term is the verbal expression of an idea.   Hence, if two different words or two different phrases express one and the same idea, these words or phrases, though differing in spelling, are one and the same term, and consequently, may be substituted for each other in a syllogism without altering it in any way.   On the other hand, in order to have three and only three *terms* in an argument, it is essential that three and only three *ideas* be expressed by the words of the argument.   If only three *words* are used, but one of them expresses one idea in one part of the argument and another idea in another part, the three words are four terms.

Rule 1, then, warns us particularly against the use of an ambiguous word in the syllogism.   The following examples violate this rule: "The bow is the foremost part of a ship; But a nod of the head is a bow; Therefore a nod of the head is the foremost part of a ship."   "Man is a species; But Cicero is a man; Therefore Cicero is a species" (cf. 18, 107).

The violation of Rule 1 is called **Quarternio Termi-
norum** or the **Fallacy of Four Terms.**

80. **Apparent Exceptions to Rule 1.** It is a point
of special importance to .remark that the syllogism is
an argument which expresses only *one* process of infer-
ence (cf. 55). When, therefore, we say that the *simple*
categorical syllogism can have only three terms, we
are contemplating the verbal expression of a *single*
process of inference. Doubtless, if an argument ex-
presses more than one process, it may contain more
than three terms. Consequently, no argument of four
or more terms, even though conveyed in three cate-
gorical propositions, can be fairly adduced as an ex-
ception to Rule 1, if it can be shown to be an elliptical
expression of more than one process.

> The horse is larger than a dog,
> The elephant is larger than a horse,
> Therefore The elephant is larger than a dog.

It is not unusual to find examples of this kind cited
by logicians as a proof that the categorical syllogism
may contain more than three terms. This argument,
they say, is valid, though it has four terms, namely,
"horse," "larger than a dog," "elephant," and "larger
than a horse." But, as a matter of fact, this argument
is not a complete expression of a process of inference.
More than that, it is an elliptical expression of *two*
processes. "The horse is larger than a dog:"—this is
not a premise of the process of inference by which we
reach the conclusion, "Therefore the elephant is larger
than a dog;" it is a *datum* from which we derive by
eduction a major premise which is not expressed, viz.

"Every thing larger than a horse is larger than a dog"
(cf. 45). This eduction is one process that is not com-
pletely stated. Again, we could not arrive at the con-
clusion, "Therefore the elephant is larger than a dog,"
unless we had some common term or basis with which
to compare the subject and predicate of the conclusion,
viz. "elephant" and "(thing) larger than a dog" (cf.
APPENDIX: *Note on Section 21*). In the original argu-
ment this common or middle term is not expressed in
two premises. Consequently, the two propositions,
"The elephant is (a thing) larger than a horse,"
"Therefore the elephant is (a thing) larger than a
dog," are in reality an enthymeme (cf. 55). The argu-
ment, fully stated, would run as follows:

> The horse is (a thing) larger than a dog,
>
> Therefore Every thing larger than a horse is (a thing)
> larger than a dog,
>
> The elephant is (a thing) larger than a
> horse,
>
> Therefore The elephant is (a thing) larger than a dog.

The middle term is "thing larger than a horse." It
is obvious that we could not have drawn the conclu-
sion in the argument as originally stated, had we not
recognized that everything larger than a horse is
larger than a dog. Cf. APPENDIX: *Note on Section 80.*

> Gold is a precious metal,
>
> This cup is plated with gold,
>
> Therefore This cup is plated with a precious metal.

There are five terms in this argument, viz. "gold,"
"precious metal," "this cup," "plated with gold," and
"plated with a precious metal." But here also there are

two processes of inference underlying the argument.
From the proposition, "Gold is a precious metal," an
unexpressed major premise is derived by means of
Eduction by Complex Conception (cf. 44). Expressed
in full, the argument would run—

<div style="text-align:center">

Gold is a precious metal,

</div>

Therefore Every thing plated with gold is plated with
a precious metal,

<div style="text-align:center">

This cup is plated with gold,

</div>

Therefore This cup is plated with a precious metal.

81. RULE II: *No term may be distributed in the*
conclusion which was not distributed in the premises.

This rule is needlessly vague. It should read, "The
minor term should not be distributed in the conclu-
sion, if it was not distributed in its premise" (cf. 78).

Rule 2 is provided for in the *dicta* of the third figure,
which allow only a particular conclusion. It is pro-
vided for in the *dicta* of the first and second figures
which warrant a conclusion only with reference to
the things ("anything") mentioned in the minor
premise.

The reason for Rule 2 is that, if the minor term were
distributed in the conclusion without having been dis-
tributed in its premise, the conclusion would be as-
serting more than was implied in the premises, and
hence it would not be logically dependent upon them.

This rule is violated in the following examples:
"All criminals should be imprisoned; But some Ameri-
cans are criminals; Therefore all Americans should
be imprisoned." "No birds are rational; But all birds
are bipeds; Therefore no bipeds are rational."

The violation of Rule 2 is called **Illicit Process of the Minor** or **Illicit Minor**.

RULE III: *The middle term must be distributed at least once in the premises.*

We commented on this rule in section 78.

82. RULE IV: *From two negative premises no conclusion can be drawn.*

Rule 4 is provided for in the *dicta* of the first and third figures, all of which require the minor premise to be affirmative. It is provided for in the first *dictum* of the second figure, which requires an affirmative major premise ("The formal object [P] . . . . which *possesses* a certain attribute"), and in the second *dictum* of the second figure, which requires an affirmative minor premise ("anything [S] which *possesses* that attribute").

This rule may be proved independently of the *dicta* as follows: If both premises are negative, the subject-object in both premises is asserted to be really distinct from the predicate-object (cf. 19, 4, 22 ad fin.). Suppose the subject-object is the same in both premises (Figure 3) :—Then we have no means of comparing the predicate-object of the major premise with the predicate-object of the minor so as to determine their identity or diversity; for both premises may be true, whether the predicate-objects are identical or different (distinct). For example, the two premises "No M is P" and "No M is S" may be true, whether S is or is not P. This is shown concretely in the following examples: "No horses are rational; But no horses are men; Therefore all men are rational." "No horses

are rational; But no horses are dogs; Therefore no dogs are rational."

What has just been said of the case in which the subject-object is the same in both premises holds true *mutatis mutandis* in the case where the predicate-object is the same in both premises (Figure 2), and also in the case where the subject-object of one premise is the same as the predicate-object of the other (Figures 1 and 4).

The violation of Rule 4 is called the **Fallacy of Two Negatives.**

83. Apparent Exceptions to Rule 4. The remark which was made in the first paragraph of section 80 should be recalled in connection with the present rule.

> No man who is not secure is happy,
> No tyrant is secure,

Therefore No tyrant is happy.

This argument is not a simple categorical syllogism; for it is an elliptical expression of *two* processes of inference, viz. an eduction and a process whose verbal expression is the simple categorical syllogism. The common term or basis which served for the comparison of the subject and predicate of the conclusion is not expressed in two premises in the argument, and therefore the argument is not a simple categorical syllogism. It is rather a double enthymeme. The conclusion was reached by first inferring either the obverted converse of the major premise (cf. 40) or the obverse of the minor. Which of these two processes is performed by a given individual depends on his mental habits or the particular way in which the argument

strikes him at the moment of presentation. The average mind would probably obvert the minor. In any case, the conclusion was reached in one of the following ways:

No man who is not secure is happy,
Therefore Everyone who is happy is secure,
No tyrant is secure,
Therefore No tyrant is happy.

The first line being omitted, the syllogism is in the second figure with "secure" as middle term.

No man who is not secure is happy,
No tyrant is secure,
Therefore Every tyrant is a man who is not secure,
Therefore No tyrant is happy.

Omitting the second line, we have a syllogism in the first figure in which the middle term is "man who is not secure."

84. RULE V: *Two affirmative premises cannot yield a negative conclusion.*

RULE VI: *A negative premise requires a negative conclusion.*

Rules 5 and 6 are provided for in the *dicta* of the second figure, both of which prescribe a negative premise and a negative conclusion. They are also provided for in the *dicta* of the first and third figures; for these *dicta* warrant a negative conclusion only when one of the premises is negative, and they prescribe a negative conclusion when one of the premises is negative, as is evident from the following words which appear in all of them: "Any attribute [P]

which is affirmed or denied . . . may be affirmed
or denied *respectively*."

Without reference to the *dicta* these rules may be
proved as follows:

Proof of Rule V: If both premises are affirm-
ative, they assert that the objects denoted by the
major and minor terms are both *identical* with the
object denoted by the middle term (cf. 19).   Hence,
by the Axiom of Identity (cf. 63) they are *identical*
with each other, and the conclusion must be *affirma-*
tive. The following example violates Rule 5: "All
negroes are black; But some Americans are negroes;
Therefore some Americans are not black."

Proof of Rule VI: If one premise is negative,
then, by Rule 4, the other premise must be affirmative.
The negative premise asserts that the object denoted
by one of the extremes (cf. 64) is *different* from the
object denoted by the middle term; the affirmative
premise asserts that the object denoted by the other
extreme is *identical* with the object denoted by the
middle term. Hence, by the Axiom of Diversity, the
objects denoted by the extremes, that is, by the major
and minor terms, are *different* from each other, and
the conclusion must be *negative*. Rule 6 is violated
in the following example: "No noble-minded men are
selfish; But some statesmen are noble-minded men;
Therefore some statesmen are selfish."

85. **Apparent Exceptions to Rule 5.** The apparent
exceptions to Rule 5 generally involve a process of
obversion in addition to the process of the categorical
syllogism. "All men are rational; This animal is

irrational; Therefore this animal is not a man." The two processes of inference of which this example is an elliptical expression are stated in full as follows:

> All men are rational,
> This animal is irrational,
> Therefore This animal is not rational,
> Therefore This animal is not a man.

Omitting the second line, we have a syllogism in the second figure.

86. **Apparent Exceptions to Rule 6.** The apparent exceptions to Rule 6 involve at least two processes of inference. "Every material that is not compound is an element; Gold is not compound; Therefore gold is an element." The complete expression of the inferences underlying this argument is as follows:

> Every material that is not compound is an element,
> Gold is not compound,
> Therefore Gold is a material that is not compound,
> Therefore Gold is an element.

With the second line omitted, the argument is a syllogism in the first figure having for its middle term "material that is not compound."

RULE VII: *A particular premise requires a particular conclusion.*

As we saw in section 78, this rule is superfluous: it is covered by Rule 2.

87. RULE VIII: *From two particular premises no conclusion can be drawn.*

Rule 8 is provided for in all the *dicta*, everyone of

which prescribes a universal premise. This may be seen from the rules of the three first figures.

For a theoretical proof of Rule 8 which is independent of the *dicta,* consult the *Note* at the end of section 73. Here it will be sufficient to prove the rule concretely.

Take, first, a case in which both premises are affirmative. The conclusion will then be affirmative (Rule 5). Let the letters P, M, S stand for "palace," "magnificent," and "statue," respectively. Arrange these terms in particular affirmative propositions in any of the figures, and it will be found that the premises are true. Nevertheless, we know that the conclusion, "Some statues are palaces," is false (cf. 65).

Take, secondly, a case in which one premise is negative. This should give us a negative conclusion (Rule 6). Let P, M, S denote "pointed," "massive," and "spear," respectively. These terms may be arranged in particular propositions according to any of the figures. Moreover, in any figure either the major or the minor premise may be negative. In every case we know that the premises are true. And yet we cannot draw the conclusion, "Some spears are not pointed;" for we know that this is not true.

88. **Apparent Exceptions to Rule 8.** In section 67 we said that in attempting to determine the rules of the categorical syllogism we supposed that "some," in the sense of "one at least," was the only sign of a particular proposition employed in the syllogism. If signs like "most" and "two-thirds" are used, Rule 8 does not apply universally; for then the proof of the rule would

not hold.   Consequently, the argument given below is
not an exception to Rule 8; for it disregards the con-
dition on which the rule was laid down.

> Most Americans are free,
> Most Americans are white,
Therefore Some white persons are free.

"Most" has the same force as the expression "at
least one more than half," and hence the conclusion
is valid.   By reason of the sign "most" we know that
the Americans who are free, referred to in the major
premise, overlap the Americans who are white, re-
ferred to in the minor.

The following *dictum* applies to syllogisms like the
one we have just been considering: "Any attribute
[P] which in most cases is affirmed or denied of the
formal object [M] of a universal idea which [M] in
the majority of cases possesses a second attribute [S]
may be affirmed or denied respectively of something
[S] having the second attribute."

CHAPTER XIII

REDUCTION OF CATEGORICAL SYLLOGISMS

89. The **reduction of a categorical syllogism** is the process of reconstructing a syllogism of the second, third, or fourth figure upon the plan of the first so as to obtain the same conclusion.

Since the general rules of the categorical syllogism, when proved apart from the *dicta,* are negative in character, not making manifest by themselves the validity of a conclusion, but putting us on our guard against certain causes of invalidity, the original purpose of reduction was to submit every mood outside the first figure to the test of the *Dictum de omni et nullo* (cf. 69). But this could not be done except by refashioning the mood according to the first figure, because the *Dictum* applies to this figure alone. However, recourse to this test is no longer necessary, for we have *dicta* which apply to all the moods of the three first figures. Nevertheless, though the reduction of syllogisms is unnecessary as a means of verification, it is a good logical exercise, because it increases our knowledge of the relations between the various terms and the various propositions which enter into the syllogism. The employment of concrete examples will show that many arguments will not fit naturally into the first figure.

Direct reduction is a reduction by means of conversion alone or by means of conversion combined with transposition of the premises.

Indirect reduction is a reduction in which the contradictory of the conclusion is shown to be inconsistent with the premises. We prove a proposition indirectly, when we show that its contradictory is incompatible with what is already held to be true (cf. 36).

90. We may illustrate *direct* reduction by the following example, in which the syllogism to be reduced is in the third figure:

> All metals are easily combined with oxygen,
> Some metals are lighter than water,
> Therefore Some things lighter than water are easily combined with oxygen.

If the minor premise is converted *simply,* the syllogism will be in the first figure.

Indirect reduction may be illustrated by means of the following syllogism in the second figure:

> All horses are quadrupeds,   *Baroco*
> Some animals are not quadrupeds,
> Therefore Some animals are not horses.

If this conclusion is not true, its contradictory ("All animals are horses") is true (cf. 36). Combining the contradictory of the conclusion with the premises, which are granted to be true, the three following propositions must be true together:

> All horses are quadrupeds,
> Some animals are not quadrupeds,
> All animals are horses.

Omitting the second proposition, we have a syllogism in the first figure:

> All horses are quadrupeds,　　　　*Barbara*
> All animals are horses,

Therefore All animals are quadrupeds.

The person who admitted the premises of the original syllogism, but denied the conclusion, would have to hold both the following propositions to be true: "Some animals are not quadrupeds," "All animals are quadrupeds;" but these propositions are inconsistent with each other, since they are contradictories. Consequently, anyone who grants the original premises must also grant the conclusion which was derived from them.

Indirect reduction is also called **Reductio ad impossibile** or **Reductio ad absurdum.** It is only by this method that we can reduce *Baroco* of the second figure and *Bocardo* of the third to the first figure so long as we retain the terms of the original syllogism.

THE MNEMONIC LINES

91. The mnemonic lines are here repeated for convenience of reference:

Barbara, Celarent, Darii, Ferio*que prioris;*
Cesare, Camestres, Festino, Baroco, *secundæ;*
Tertia, Darapti, Disamis, Datisi, Felapton,
Bocardo, Ferison, *habet; Quarta insuper addit*
Bramantip, Camenes, Dimaris, Fesapo, Fresison.

De Morgan speaks of these lines as "the magic words by which the different moods have been denoted for many centuries, words which I take to be

more full of meaning than any that ever were made."

The mnemonic lines inform us in detail how each mood of figures 2, 3, and 4 is to be reduced to figure 1.

The consonants, b (not initial), d (not initial), l, n, r, t, are the only letters that have no meaning.

The *vowels,* a, e, i, o, indicate the quality and quantity of the propositions in the mood. Thus, the following syllogism in the fourth figure is represented by *Camenes:*

A   All animals are sentient things,
E   No sentient things are plants,
E   Therefore No plants are animals.

The *initial letters, B, C, D, F,* appearing in figures 2, 3, and 4, signify that any given mood in those figures is to be reduced to that mood of the first figure which has the same initial letter. *Camestres,* for example, is reduced to *Celarent:*

All P is M   No M is S
No S is M   All P is M
Therefore No S is P   Therefore No P is S
                                Therefore No S is P

All unselfish men are lovable,
No cruel men are lovable,
Therefore No cruel men are unselfish.
Reduced to *Celarent,* this syllogism reads:

No lovable men are cruel,
All unselfish men are lovable,
Therefore No unselfish men are cruel,
Therefore No cruel men are unselfish.

The use of arrows may help to make the reduction clearer, as follows:

All P is M ⟶⟶⟶⟶⟶⟶⟶⟶⟶⟶⟶⟶⟶⟶⟶⟶⟶⟶⟶⟶⟶⟶⟶⟶ No M is S
No S is M ⟶⟶⟶⟶⟶⟶⟶⟶⟶⟶⟶⟶⟶⟶⟶⟶⟶⟶⟶⟶⟶⟶⟶⟶ All P is M
                                                                       ↓
No S is P ⟵⟵⟵⟵⟵⟵⟵⟵⟵⟵⟵⟵⟵⟵⟵⟵⟵⟵⟵⟵⟵⟵ No P is S
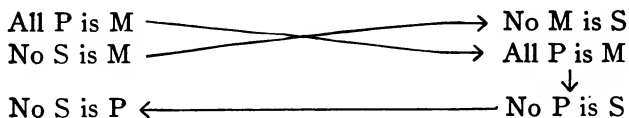
s (in the middle of a word) signifies that the premise immediately preceding it is to be converted *simply*. Thus, in the reduction of *Camestres* we converted the minor premise *simply*.

s (at the end of a word) means that the conclusion of the *new* syllogism is to be converted *simply,* so that we may obtain the original conclusion. This is also illustrated in the foregoing reduction of *Camestres.*

p (in the middle of a word) indicates that the premise immediately preceding it is to be converted *per accidens.* This is done, for example, in reducing *Felapton to Ferio:*

No M is P	No M is P
All M is S	Some S is M
Therefore Some S is not P	Therefore Some S is not P

No metals are organic,
All metals combine with oxygen,
Therefore Some things that combine with oxygen are
    not organic.
Reduced to *Ferio,* the syllogism reads:
No metals are organic,
Some things that combine with oxygen are
    metals,
Therefore Some things that combine with oxygen are
    not organic.

p (at the end of a word) shows that the *new* conclusion is to be converted *per accidens* in order to give us the original conclusion. This, for example, is what happens in the reduction of *Bramantip* to *Barbara:*

All P is M   All M is S
All M is S   All P is M
Therefore Some S is P   Therefore All P is S
                                         Therefore Some S is P

    All emperors are men,
    All men are mortal,
Therefore Some mortal beings are emperors.
Reduce this syllogism to *Barbara,* and it reads:

    All men are mortal,
    All emperors are men,
Therefore All emperors are mortal,
Therefore Some mortal beings are emperors.

m signifies that the premises are to be transposed *(metathesis præmissarum)*, that is, that the major and minor premises are to become minor and major premises respectively. This is illustrated in the foregoing reduction of *Bramantip,* as well as in that of *Camestres.*

c indicates that the mood is to be reduced *indirectly,* and that in the process the premise immediately preceding this letter is to be replaced by the contradictory of the conclusion.

CHAPTER XIV

THE PURE HYPOTHETICAL SYLLOGISM
AND OTHER TYPES OF ARGUMENT

92. The pure hypothetical syllogism is the only type of argument in the present chapter which has not been explained in the chapters preceding, at least in its essential features. If we regard the disjunctive proposition as a variant of the hypothetical, we shall find that the dilemma is a modification of the mixed or of the pure hypothetical syllogism. The remaining types of argument either abridge or modify those already discussed or else they combine several of them into one.

THE PURE HYPOTHETICAL SYLLOGISM

93. The **pure hypothetical** or **pure conditional syllogism** is a syllogism which proves a hypothetical conclusion by means of two hypothetical premises in which the antecedent and the consequent of the conclusion are compared separately with a third potential judgment (cf. 46). For example—

> If A is B, C is D,
> If E is F, A is B,
> Therefore If E is F, C is D.

The process of inference expressed by the categorical syllogism has for its *remote matter* the formal

objects of three ideas, one of which affords a basis of comparison for determining the objective identity or diversity of the two others. The remote matter of the process of inference expressed by the pure hypothetical syllogism are the formal objects of three potential judgments, one of which affords a basis of comparison for determining the *logical dependence* of one of the others upon the remaining one. In the categorical syllogism the subject and predicate of the conclusion (that is, the minor and major terms), besides appearing in the conclusion, appear each once in the premises, and the middle term appears twice in the premises. In the pure hypothetical syllogism the antecedent and consequent of the conclusion, besides appearing in the conclusion, appear each once in the premises; and there is a third potential judgment ("A is B" in the foregoing example) which appears twice in the premises and is the common basis of comparison for the antecedent and consequent of the conclusion. Hence, the antecedent and consequent of the conclusion correspond in general to the minor and major terms respectively, and the potential judgment which appears only in the premises, to the middle term. Cf. APPENDIX: *Note on Section 93*.

We may, therefore, determine the "figure" of a hypothetical syllogism as we did that of a categorical, our criterion being the relative positions of the common potential judgment in the premises.

The *dicta* of the three figures of the pure hypothetical syllogism will be found at the end of the Appendix.

In Figure 1 the common potential judgment is antecedent in the major premise and consequent in the minor.

In Figure 2 it is consequent in both premises.

In Figure 3 it is antecedent in both premises.

If the minor premise be denominated affirmative or negative, according as the potential judgment in its consequent is the same as it was in the major premise or the contradictory of what it was, the following rules may be laid down for these figures:

In Figure 1 the minor premise must be affirmative.
In Figure 2 the minor premise must be negative.
In Figure 3 the conclusion must be particular.

The figures are illustrated concretely in the following examples:

Fig. 1: If the man is not guilty, he should be acquitted,

If he was away from home, he is not guilty,

.Therefore If he was away from home, he should be acquitted.

Fig. 2: If the animal is rational, it is a man, ·

If the animal is a quadruped, it is not a man,

Therefore If the animal is a quadruped, it is not rational.

Fig. 3: If a man is a general, he is brave,

If a man is a general, he is intelligent,

Therefore Sometimes if a man is intelligent, he is brave.

Note.—There are syllogisms in which a hypothetical or a disjunctive conclusion is derived from premises, one of which is categorical. For example—

> If A is B, it is C,
> D is A,
>
> Therefore If D is B, it is C.
>
> A is either B or C,
> D is A,
>
> Therefore D is either B or C.

No special names have been assigned to these types of syllogism. The name "hypothetico-categorical" has sometimes been employed to designate the mixed hypothetical syllogism. If we interpret this name as signifying a syllogism with a hypothetical premise and a categorical conclusion, there will be a certain appropriateness in designating the two foregoing types of syllogism respectively by the names "categorico-hypothetical" and "categorico-disjunctive."

THE DILEMMA

94. The **dilemma** is an argument in which the major premise is a compound hypothetical proposition, and the minor premise, a disjunctive proposition alternatively positing the antecedents or sublating the consequents of the major.

The force of the constructive dilemma is more striking when the disjunctive premise is stated first.

Strictly speaking, the word "dilemma" implies only two alternants; but it is commonly used even when there are three or more.

The **constructive dilemma** is one in which the minor

premise alternatively *posits the antecedents* of the major.

The **destructive dilemma** is one in which the minor premise alternatively *sublates the consequents* of the major.

In the *constructive dilemma* the major premise must have at least two different *antecedents;* otherwise, the minor premise could not posit *alternatively.* The consequents may be either the same or different. When the consequents are the same, the dilemma is **simple constructive;** when they are different, it is **complex constructive.** In the simple constructive dilemma the conclusion is a simple categorical proposition positing the consequent; in the complex constructive the conclusion is a disjunctive proposition positing the consequents alternatively.

(1) **Simple constructive dilemma—**

If A is B, E is F; and if C is D, E is F;
But either A is B or C is D;

Therefore E is F.

(2) **Complex constructive dilemma—**

If A is B, E is F; and if C is D, G is H;
But either A is B or C is D;

Therefore Either E is F or G is H.

In the *destructive dilemma* the major premise must have at least two different *consequents.* The antecedents may be either the same or different; and the dilemma will be **simple destructive** or **complex destructive** accordingly.

(1) **Simple destructive dilemma—**

If A is B, C is D; and if A is B, E is F;
But either C is not D or E is not F;

Therefore A is not B.

(2) **Complex destructive dilemma—**

>>If A is B, E is F; and if C is D, G is H;
>>But either E is not F or G is not H;

Therefore Either A is not B or C is not D.

Concrete examples of the dilemma—

>>If Logic furnishes useful principles, it is worthy of study; and if it trains the mind, it is worthy of study;
>>But it either furnishes useful principles or trains the mind;

Therefore Logic is worthy of study.

>>If Æschines joined in the public rejoicings, he is inconsistent; and if he did not, he is unpatriotic;
>>But he either joined in them or he did not;

Therefore He is either inconsistent or unpatriotic.

>>If a man is a leader, he is attentive to details; and if he is a leader, he has a strong influence upon others;
>>But either George will not be attentive to details or he will not have a strong influence upon others;

Therefore George will not be a leader.

>>If the man were intelligent, he would perceive the mistake; and if he were honest, he would acknowledge it;
>>But either he does not perceive the mistake or he will not acknowledge it;

Therefore He is not intelligent or he is not honest.

The *simple constructive* and the *simple destructive* dilemmas may sometimes be stated as follows with a

gain in conciseness and force:

   (1) **Simple constructive—**

> If Logic either furnishes useful principles
> or trains the mind, it is worthy of study;
> But it either furnishes useful principles or
> trains the mind;

Therefore It is worthy of study.

   (2) **Simple destructive—**

> If a man is a leader, he both attends to de-
> tails and influences other men;
> But George either will not attend to details
> or he will not influence other men;

Therefore George will not be a leader.

Note.—Arguments like the following are also fre-
quently regarded as dilemmas:

> If A is B, C is either D or E,
> But C is neither D nor E,

Therefore A is not B.

RULES OF THE DILEMMA

95. RULE I: *All the pertinent alternants must be
stated in the disjunctive premise.*

If a boy, upon quitting college, has mastered the
branches of the college course, he has a well
equipped mind; and if he has not mastered them,
he has not profited by the course.

But every boy, upon quitting college, has mastered
the branches of the college course or he has not
mastered them.

Therefore every boy, upon quitting college, has a
well equipped mind or he has not profited by the
course.

In this argument the disjunctive minor premise does not exhaust all the pertinent alternants. The disjunctive premise should read: "But every boy, upon quitting college, has mastered *all* the branches of the college course or he has mastered *none* of them or he has mastered *some* of them."

When we add an alternant to a dilemma that has been directed against us, we are said to **escape between the horns of the dilemma.**

It is to be observed that the number of alternants in the disjunctive premise determines the number of elements in the compound hypothetical premise.

RULE II: *The logical dependence of each consequent upon its antecedent in the major premise must be real and manifest.*

> If I devote myself to my worldly interests, I shall lose my soul; and if I devote myself to the interests of my soul, I shall ruin the position of my family.
>
> But I must either devote myself to my worldly interests or to those of my soul.
>
> Therefore I shall either lose my soul or ruin the position of my family.

In reply to this argument we might say that it does not follow that a man will lose his soul because he devotes himself to his worldly interests; nor does it follow that he will ruin the position of his family because he devotes himself to the interests of his soul.

Answering a dilemma in this way is called **taking the dilemma by the horns.**

It may be observed in addition that the disjunctive premise in the last example omits the third alternant of attending to the interests of both.

RULE III: *The hypothetical premise must state all the pertinent consequents warranted by each antecedent.*

An Athenian mother, attempting to dissuade her son from entering public life, used the following argument:

> If you say what is just, men will hate you; and if you say what is unjust, the gods will hate you.
> But you must say what is just or what is unjust.

Therefore You will be hated.

The son answered as follows:

> If I say what is just, the gods will love me; and if I say what is unjust, men will love me.
> But I must say what is just or what is unjust.

Therefore I shall be loved.

The original argument failed to state all the pertinent consequents that followed from each of the antecedents in the hypothetical premise. The omitted consequents were supplied in the rejoinder and the others left out.

When we answer a dilemma in this way we are said **to retort the dilemma.**

96. **Some Famous Dilemmas and Sophisms.** The following sophistical arguments have come down to us from ancient Greece:

The "Litigiosus." Protagoras, the Sophist, is said to have agreed to teach Euathlus the art of pleading, the stipulation being that Euathlus should pay one-half of the fee when fully instructed, and the other half when he won his first case in court. Euathlus put off practising his profession, and finally he was sued by Protagoras for the rest of the fee. The following argument was advanced by Protagoras:

> If Euathlus loses this case, he must pay me, by the judgment of the court; and if he wins it, he must pay me, by the terms of the contract.
>
> But he must either win it or lose it.
>
> Therefore he must pay me in any case.

Euathlus retorted as follows:

> If I win this case, I ought not to pay, by the judgment of the court; and if I lose it, I ought not to pay, by the terms of the contract.
>
> But I must either win it or lose it.
>
> Therefore I ought not to pay in any case.

Comment: Protagoras and Euathlus were both guilty of sophistical reasoning. Both started by pronouncing the decision of the court to be binding; and then they immediately and unconcernedly implied that it was not binding by appealing to the contract. Now, either the litigants regarded the decision of the court as binding, or they did not. If they regarded it as binding, they had no right to appeal from it to the contract. If they did not regard it as binding, they were dishonest in appealing to it at all in their argument.

The "Liar." Epimenides, the Cretan, says that all Cretans are liars.

> If Epimenides' statement is not true, he is a liar; and if it is true, he is a liar, for he is a Cretan.
>
> But his statement is either true or not true.
>
> Therefore he is a liar.
>
> But since he is a liar, his statement is not true that all Cretans are liars.
>
> Therefore some Cretans are not liars.
>
> But since some Cretans are not liars, Epimenides is not necessarily a liar because he is a Cretan.
>
> Therefore we may accept his statement that all Cretans are liars. And so on.

Comment: There is no difficulty, if the statement of Epimenides means that Cretans *generally* utter falsehoods. But if we suppose it to be a fact that Cretans never speak the truth, we cannot suppose a Cretan to say so without involving ourselves in a contradiction. The two propositions—"Cretans never speak the truth" and "Epimenides, the Cretan, said so"—cannot be true together (*Keynes*).

Argument of Zeno against Motion:

> If an arrow moves, it must move either in the place where it is or in the place where it is not.
>
> But it cannot move in the place where it is, else it would not be there; nor can it move in the place where it is not, for it is not there to move.
>
> Therefore an arrow cannot move.

Comment: A body can be in a place in two ways: first, it may be where it was before, and this is *rest;* secondly, it may be in a place in which it was *not*

before and from which it immediately recedes, that is, it does not remain where it is, and this is *motion*. Hence, the arrow not only moves from the place where it was to a place where it is not; it also moves in the place where it is, but where it is *transiently;* that is, it does not *remain* in the place where it is, but acquires another place, in which again it is, and from which again it recedes without remaining there; and so on, till it remains in the place it occupies, and this is to rest.

If in the major premise of Zeno's argument we substitute for "move" words which express its meaning clearly and unambiguously, we shall see immediately that the premise is not true or, at least, that it is open to two interpretations. Thus: "If an arrow occupies in successive instants of time successive positions in space, it must occupy those successive positions either where it is or where it is not." In order to get rid of the ambiguity in the major premise, we should employ three alternants instead of two, viz. "Either where it is permanently (i.e. where it remains) or where it is transiently (i.e. where it is momentarily but does not remain) or where it is not."

THE ENTHYMEME

97. The **enthymeme** is an abridged syllogism, one of the premises or the conclusion being omitted.

The arguments employed in speaking and writing are usually in the form of an enthymeme. This is also the form frequently assumed by the fallacious argument, which often gains in plausibility by failing in explicitness.

According as the major premise, the minor premise, or the conclusion is omitted, the enthymeme is said to be of the first, the second, or the third order.

The syllogism: All bullies are cowards,
                       But Verres is a bully,
            Therefore Verres is a coward.

The first order: Verres is a bully,
            Therefore He is a coward.

The second order: All bullies are cowards,
            Therefore Verres is a coward.

The third order: All bullies are cowards,
                       And Verres is a bully.

For purposes of rhetorical effect, the enthymeme of the third order is much preferable to the complete syllogism.

Like the categorical syllogism, the hypothetical syllogism often appears in abbreviated form; e.g. "There is a just God; Therefore the heroic virtues of the Martyrs have been rewarded."

THE POLYSYLLOGISM

98.   The **polysyllogism** is a series of syllogisms so connected that the conclusion of one is a premise of another.

A **prosyllogism** is a syllogism the conclusion of which is a premise of another syllogism.

An **episyllogism** is a syllogism a premise of which is the conclusion of another syllogism.

Any intermediate syllogism in a polysyllogism is an episyllogism with reference to the syllogism immediately preceding, and a prosyllogism with reference to the one immediately following. ·

The following example illustrates the polysyllogism:

> A man who desires more than he has is discontented,
>
> An avaricious man desires more than he has,

Therefore An avaricious man is discontented.

> A miser is an avaricious man,

Therefore A miser is discontented.

> Balbus is a miser,

Therefore Balbus is discontented.

THE SORITES

99. The **sorites** is a polysyllogism in which all the conclusions except the last are omitted, the premises being so arranged that any two successive premises have a common term or a common potential judgment.

The two common forms of the sorites are the Aristotelian and the Goclenian. The Aristotelian sorites was the only form mentioned in works on Logic before the sixteenth century. It is called Aristotelian to distinguish it from the Goclenian sorites. Rudolf Goclenius of Marburg (1547 to 1628) was the first to call attention to the Goclenian form; hence its name.

The **Aristotelian sorites** is one in which the first premise contains the *subject* of the conclusion and every term common to two successive premises appears first as predicate and then as subject.

The **Goclenian sorites** is one in which the first premise contains the *predicate* of the conclusion and every term common to two successive premises appears first as subject and then as predicate.

Aristotelian Sorites	Goclenian Sorites
All A is B	All D is E
All B is C	All C is D
All C is D	All B is C
All D is E	All A is B
Therefore All A is E	Therefore All A is E

In the Aristotelian sorites the first premise is a *minor* premise; for it contains the *subject* of the conclusion. The remaining premises are *major* premises.

In the Goclenian sorites the first premise is a *major* premise; for it contains the *predicate* of the conclusion. The remaining premises are *minor* premises.

When the argument of the *Aristotelian* sorites is stated in full, all the suppressed conclusions appear as *minor* premises in successive syllogisms. Thus, the foregoing Aristotelian sorites is an abridged expression of the three following syllogisms:

(1)   All B is C,
                        All A is B,
        Therefore All A is C.

(2)   All C is D,
                        All A is C,
        Therefore All A is D.

(3)   All D is E,
                        All A is D,
        Therefore All A is E.

When the argument of the *Goclenian* sorites is stated in full, all the suppressed conclusions appear as *major* premises in successive syllogisms. Thus, the foregoing Goclenian sorites is an abridged expression of the three following syllogisms:

(1)   All D is E,
                       All C is D,
         Therefore All C is E.
(2)   All C is E,
                       All B is C,
         Therefore All B is E.
(3)   All B is E,
                       All A is B,
         Therefore All A is E.

The following is a concrete example of the *Aristotelian* sorites:

          Balbus is a miser,
          A miser is an avaricious man,
          An avaricious man desires more than he has,
          A man who desires more than he has is discontented,
Therefore Balbus is discontented.

The constituent syllogisms of the foregoing sorites are as follows:

(1)   A miser is an avaricious man,
               Balbus is a miser,
     Therefore Balbus is an avaricious man.
(2)   An avaricious man desires more than he has,
               Balbus is an avaricious man,
     Therefore Balbus desires more than he has.
(3)   A man who desires more than he has is discontented,
               Balbus desires more than he has,
     Therefore Balbus is discontented.

The following example illustrates the *Goclenian* sorites:

> A man who desires more than he has is discontented,
>
> An avaricious man desires more than he has,
>
> A miser is an avaricious man,
>
> Balbus is a miser,

Therefore Balbus is discontented.

This sorites is resolvable into the following syllogisms:

(1)
> A man who desires more than he has is discontented,
>
> An avaricious man desires more than he has,

Therefore An avaricious man is discontented.

(2)
> An avaricious man is discontented,
>
> A miser is an avaricious man,

Therefore A miser is discontented.

(3)
> A miser is discontented,
>
> Balbus is a miser,

Therefore Balbus is discontented.

The following is an example of the **pure hypothetical sorites**:

> If Balbus hoards his gold, he is a miser,
>
> If he is a miser, he is avaricious,
>
> If he is avaricious, he desires more than he has,
>
> If he desires more than he has, he is discontented,

Therefore If Balbus hoards his gold, he is discontented.

100. It will have been observed that the syllogisms into which both the Aristotelian and the Goclenian sorites may be resolved are all in the *First Figure*. Consequently, the rules of both forms of sorites will be deducible from the rules of the first figure.

Rules of the Aristotelian Sorites. It is to be noticed that in the Aristotelian sorites all the premises except the first are *major* premises; the first premise and all the suppressed conclusions are *minor* premises.

RULE I: *Every premise, except the first, must be universal.*

RULE II: *Every premise, except the last, must be affirmative.*

Proof of Rule 1: Every premise in the Aristotelian sorites, except the first, is a *major* premise: hence, every premise, except the first, must be universal; for the major premise in the first figure must be universal (cf. 71):

The first premise and all the suppressed conclusions are *minor* premises: therefore, the first premise may be particular.

Proof of Rule 2: Every premise, except the last, must be affirmative; for if any other than the last were negative, it would yield a negative conclusion (cf. 84). This negative conclusion would become a minor premise, and the minor premise in the first figure cannot be negative.

The last premise is the major premise of the last syllogism, and therefore it may be negative.

Rules of the Goclenian Sorites. In the Goclenian sorites all the premises, except the first, are *minor*

premises; the first premise and all the suppressed conclusions are *major* premises.

RULE I: *Every premise, except the last, must be universal.*

RULE II: *Every premise, except the first, must be affirmative.*

Proof of Rule 1: Every premise, except the last, must be universal; for if any other than the last were particular, it would yield a particular conclusion. This particular conclusion would become a major premise, and the major premise in the first figure cannot be particular.

The last premise is the minor premise of the last syllogism, and therefore it may be particular.

Proof of Rule 2: Every premise in the Goclenian sorites, except the first, is a *minor* premise; hence, every premise, except the first, must be affirmative; for the minor premise in the first figure must be affirmative.

The first premise and all the suppressed conclusions are *major* premises: therefore, the first premise may be negative.

THE EXPOSITORY SYLLOGISM

101. The **expository syllogism** is a syllogism in which the middle term is singular. The major and minor terms may or may not be singular. The expository syllogism is illustrated in the following example:

<div align="center">

Cicero was a Roman,

Cicero was an orator,

Therefore Some orator was a Roman.

</div>

This type of syllogism is called *expository,* because it exposes the matter, as it were, before our eyes.

The expository syllogism may be constructed according to any figure; but it is commonly found in the third, where the middle term is subject in both premises. It is unusual to meet a proposition with a singular term as predicate, unless the subject too is singular.

Our warrant for employing a singular term as middle term in the first and third figures is that in argument a proposition with a singular term as subject is equivalent to a universal proposition (cf. 24). If a singular term is middle term in the second figure, the validity of the syllogism will sometimes be determined only by a knowledge of its *matter.*

CHAPTER XV

THE PREDICABLES AND THE CATEGORIES

102. THE PREDICABLES. One of the busiest functions of the human mind is the detection of points of resemblance and dissimilarity between the multitudinous objects which fall under its observation. At first, these objects are apparently a disordered and chaotic mass. When confronted with this medley of disorganized materials, the mind grows restive, and strives to discover some principle of order. By scrutinizing, comparing, and analyzing the maze of materials before it, the mind comes to recognize that the objects are not absolutely diverse from each other, that many of them have certain features in common which are not shared by others, while large numbers of the latter also have their own points of agreement. The mind makes a mental note of the characteristics or, at least, of some important characteristic possessed in common by a number of the objects, and these objects it arranges in a group. It then proceeds in a similar way with the remaining materials. But the mind is not content to leave the groups in isolation with no point of contact between them. It endeavors to discover some connecting link, proximate or remote, between group and group. It is for this reason that, in forming the groups, the mind usually

148

fixes upon an attribute or aspect which the objects it
is arranging in a group have in common with other
objects. In this way the original chaos is gradually
converted into a system. This process of systematic
grouping is called **Classification.**

When a number of objects offer themselves for
classification, they confront us with a vast and diversi-
fied array of attributes. Thus, in the race of men are
found such various attributes as sentient, rational,
capable of laughing, white, learned, strong, warm-
blooded, vertebrate, mortal, and so on. The logician
arranges all the attributes in the objects under five
heads, called the Predicables, the principle of arrange-
ment being the relation which the attributes severally
bear to the purpose of the classification. The five
heads are Genus, Specific Difference, Species, Prop-
erty, and Accident. The predicables are defined as
follows:

The **predicables** are the attributes or aspects of an
object arranged according to their fitness or unfitness
to serve the purpose of a given classification.

The fitness of an attribute to serve the purpose of
a given classification is generally measured by the
amount or depth of information which, consistently
with the framing of the class, it conveys concerning
the objects to be classified. At times the fitness of an
attribute to serve this purpose is determined by con-
siderations of convenience or symmetry. The predi-
cables may also be defined as a division of the attri-
butes of an object which must be kept in view and of
those which should be disregarded in the process of a
given classification.

The attributes which must be kept in view are called **essential**: they are the Genus and the Specific Difference.

The attributes which should be disregarded are called **non-essential**: they are the Property and the Accident.

For the purpose of illustrating the different predicables, we will suppose that we are classifying meh, and that we wish to connect them with objects in the visible world.

The **genus** is an attribute which the objects to be classified have in common with other objects and which best serves the end of the classification. Thus, "animal" is the genus of man.

The **specific difference** is an attribute which is found in all the objects to be classified, but not in the other objects having the generic attribute, and which again best serves the end of the classification. For example, "rational" is the specific difference of man.

The genus and specific difference together constitute the Species or Essence of an object with reference to the particular classification; hence—

The **species** or **essence** of an object in a particular classification is its genus and specific difference in that classification. The species of man, for instance, is "rational animal."

N. B.—There is a special sense assigned to "species" or "essence" which has the sanction of immemorial usage, namely, it is the sum of the most fundamental attributes or aspects of an object. When "species" is taken in this sense, then any aspect over and above the most fundamental attributes is at the utmost a property.

A **property** is an attribute which is found in all the objects to be classified and not elsewhere, but is not best suited to the end of the classification. Thus, "capable of laughing" is a property of man.

An **accident** is an attribute which is not permanently present in the objects to be classified or is found in only some of them. For example, "sleeping" and "white" are accidents of man.

The accident which has just been defined is commonly called the **separable accident**. It is called separable, because it is not necessarily found in all the objects to be classified. There is another accident, called *inseparable*, which is defined as follows:

An **inseparable accident** is an attribute which is found in all the objects to be classified and in other objects as well, but is not best suited to the end of the classification. For example, "warm-blooded" and "vertebrate" are inseparable accidents of man.

103. The genus and specific difference must together suffice to mark off the objects to be classified from all other objects. If any one of several aspects is capable of combining with a given genus for the accomplishment of this purpose, that aspect is usually best suited to the end of the classification which either implies the others or is more fundamental than they. Thus, "rational," in the case of man, is more fundamental than "risible." What has just been said of the aspect which is to be employed for the specific difference must also be applied to the aspect which is to be selected for the genus. For example, "rational corporeal substance" and "rational organism" both suffice to mark off man

from all other individual objects; but "corporeal sub-stance" and "organism" are both implied in "animal;" hence, "animal" is best suited as a genus to the end of the classification.

In classification our selection of an attribute to serve as genus will be primarily determined by the world of objects with which we wish to connect the individuals to be classified. Suppose, for example, that men are the subject-matter of our classification. If we wish to connect them with the material creation, we shall select "animal" for our genus, and "rational" for our specific difference. If we wish to connect them with the spiritual world, our genus will be "rational being" or "rational substance," and our specific difference will be "sentient." Thus, either "rational animal" or "sentient rational being" is the species or essence of a man considered as a man. "Man gifted with the power of persuasion in public speech" is the species or essence of a man considered as an orator.

The attributes of an individual which have been selected as genus and specific difference are not com-monly called the species or essence of the individual when it is considered in itself, but only when it is viewed as a member of a particular class. Take, for example, an individual who is an orator. The power of persuasion in public speech is not essential to him considered in himself; but it is essential to him when viewed as belonging to the class of orators.

"Risible" and "having the power of speech" are *properties* of an individual, when viewed as a member of the class of men; but they are *accidents,* when the individual is viewed as a member of the class of ani-

mals. "Sleeping" is an *accident* of an individual, when regarded as belonging to the class of men; but it is *essential*, when the individual is regarded as belonging to the class of beings that are asleep.

"Rational animal" is the species of man, considered as man and as connected with the material creation. If we wish to widen the class, so as to include in it beasts, birds, fishes, etc., we shall have to drop the specific difference "rational," select for our genus an aspect which characterizes men and beasts, etc., and also the objects with which we wish to connect them, and then fix upon an aspect as specific difference which marks them off from the other objects. "Organism" will serve our purpose as genus, and "sentient" as specific difference. The species, then, is "sentient organism."

104. The **proximate genus** of an individual is the genus which in a given order implies all the other genera in the same order. For example, "animal," in the same order with "substance," and as connecting man with the material creation, is the proximate genus of man.

The **supreme genus** of· an individual is the genus which in a given order implies none of the other genera in the same order. For example, "substance," in the same order with "animal," is the supreme genus of man.

A **subaltern genus** is a genus intermediate between the proximate and the supreme genus. For example, in the same order with "animal" and "substance," "organism" is a subaltern genus of man. The supreme genus and all subaltern genera are called **Remote Genera.**

Of the ideas representing the genera of the same order the idea which represents the proximate genus has the narrowest extension, and the idea which represents the supreme genus has the widest.

105. THE CATEGORIES.   Aristotle maintained that all genera whatever and all predicates are included in one or other of ten great orders.   The supreme genus in each of these orders he called a **Category.**   In Latin the categories are called **Prædicamenta.**

The **categories,** then, according to Aristotle, are ten supreme genera under which may be grouped all genera and all predicates whatever.

The following is the list of the categories: Substance, Quantity, Quality, Relation, Activity, Passivity, Place, Time, Posture, Apparel.

The answer to any question that can be put concerning an individual will, according to Aristotle, fall under one of these categories.   This may be illustrated as follows:

SUBSTANCE: *What* is Tom Brown?   A man.

QUANTITY: How *tall* is he?   Six feet.

QUALITY: What *kind* of a man is he?   White.

RELATION: How is he *related* to Peter Brown? His son.

ACTIVITY: What is he *doing?*   Playing tennis.

PASSIVITY: What is he *undergoing?*   Defeat and ridicule.

PLACE: *Where* is he?   Behind the college.

TIME: What is the *time at which* he is playing? Three o'clock.

POSTURE: What is his *posture?* Upright, with legs apart.

APPAREL: What is he *wearing?* A tennis shirt and flannel trousers.

106. The **Tree of Porphyry** is an outline of a descending series of genera and species, beginning with Substance and ending with Man. The outline is as follows:
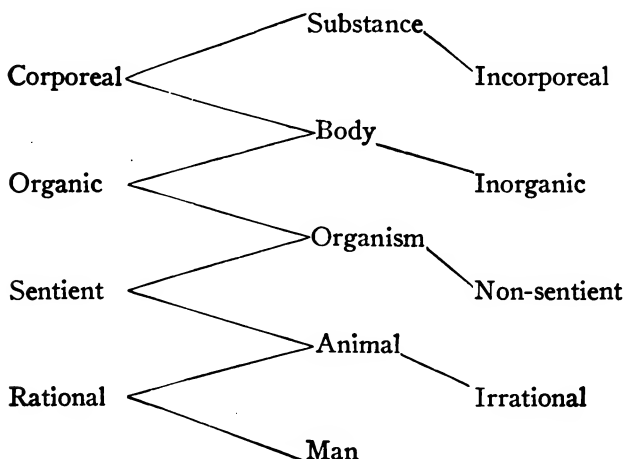
107. The ideas representing the genus, specific difference, species, property, and accident, when referring to objects outside the mind, are direct universal ideas (cf. 5). As a direct universal idea is predicated of an individual, but is not predicated *as* universal, so the idea of the genus or the species may be predicated of an individual, but it cannot be predicated *as* generic or specific. Thus, we may say, "John is an animal,"

and "John is a rational animal."   But we cannot say, "John is a species," any more than we can say, "John is a quality."   Some authors say that the predicables are reflex, and the categories, direct universal ideas. But these authors also generally call the categories the supreme genera.   If a genus is a reflex universal idea and the categories are the supreme genera, then the categories, too, are reflex universal ideas.   It is more correct to say that the ideas representing the predicables, being direct universal ideas, may become formal objects of reflex universal ideas.

CHAPTER XVI

LOGICAL DIVISION

108. **Logical division** is the resolution of a class into the various groups or sub-classes which compose it. It may also be defined as the resolution of a genus into its constituent species.

The class or genus which is resolved is called the **Totum Divisum.**

The sub-classes or species into which the *totum divisum* is resolved are called the **Dividing Members** or **Logical Parts.**

It is to be observed that in logical division the *totum divisum* is resolved into sub-classes or species, not into the *individuals* which belong to a species. The resolution of a species or sub-class into the individuals belonging to it is called **Enumeration.**

The **basis of division** is an attribute which, being variously modified in the dividing members or being present in some and lacking in others, is the ground or principle upon which the division proceeds. Thus, by selecting the size of the largest angle as our basis of division, we may divide triangles into obtuse-angled, right-angled, and acute-angled. Again, if the number of equal sides be taken as the basis, triangles may be divided into equilateral, isosceles, and scalene. Thus, the same *totum divisum* is divided in various

157

ways according to the basis of division which is selected.

Co-divisions are two or more divisions of the same class or genus, each division proceeding upon a different basis.

Co-ordinate species are the species or sub-classes derived from a genus by one process of division.

Each species or sub-class into which a genus is divided may itself be treated as a genus, and thus divided into narrower species or sub-classes. When this is done, the process is called **Subdivision.** Thus, the proposition may be divided into the categorical and the hypothetical, according as it expresses a judgment or not; then the categorical proposition may be divided into affirmative and negative, according as it asserts or does not assert the objective identity of two ideas.

Dichotomy or **division by dichotomy** is the division of a class or genus into two sub-classes or species, the basis of division being an attribute which is present in one species and absent from the other. Thus, triangles may be divided into those which are equilateral and those which are not equilateral. The Tree of Porphyry is an example of division and subdivision by dichotomy.

109. **Physical division** or **partition** is the resolution of an individual thing into the physical parts which compose it. For example, a sword may be physically divided into blade and hilt. On the other hand, the division would be logical, if swords were divided into straight and curved, or again into long and short.

An important point of difference between logical and physical division is that the *totum divisum* or genus in logical division can be predicated of all the dividing members or species and of every individual belonging to each dividing member; but the individual thing cannot be predicated of any of its physical parts. Thus, we may say, "Every man is an animal," "Every horse is an animal;" but we cannot say, "The hilt is a sword" or "The soul is a man."

Because the dividing members or species may stand as subject in an affirmative proposition in which the *totum divisum* or genus is predicate, they are sometimes called the **Subjective Parts** of the genus.

Metaphysical division or **mental distinction** is the mental division of an object into its various attributes. We have an example of metaphysical division, when we divide man into rational, sentient, organic, corporeal, warm-blooded, mortal, etc. It is the metaphysical division of objects which furnishes us with the basis of logical division.

Verbal division or **verbal distinction** is the division of an ambiguous word into its several meanings. Thus, we distinguish between palm in the sense of a tree, and palm as the hollow inner surface of the hand.

RULES OF LOGICAL DIVISION

110. The rules which are usually laid down for avoiding confusion in logical division are as follows:

RULE I: *The dividing members must be mutually exclusive as regards the basis employed in the division.*

This rule means that, as regards the basis employed in the division, none of the species must be included in another. We should offend against this rule, if we were to divide magazines into scientific, literary, and mathematical; for mathematical magazines are included in scientific magazines. A violation of this rule is called a **Cross-division**. It is to be observed, however, that if we select the maximum size of an angle as our basis of division and divide triangles into obtuse-angled, right-angled, and acute-angled, we have a perfect division with reference to the particular basis we have employed, though examples of all three kinds could be found in one species if the basis of division were "having two sides equal."

RULE II: *The dividing members must together be co-extensive with the class or genus which is divided.*

This rule may be violated either by defect or by excess, that is, by omitting a species or dividing member or by including something that is not a species of the genus which is divided. If we were to divide mankind on the basis of color into white men, red men, and black men, we should sin by defect; for we have omitted the yellow men and the brown men. If metals were divided into solid, liquid, and gaseous, the division would be too wide; for there are no gaseous metals.

RULE III: *Each process of division must have only one basis.*

If this rule is violated, the result may be that some objects are included in more than one species. If men were divided into Americans, Englishmen, Irishmen, Frenchmen, Germans, mechanics, and artists, two

bases would be employed, namely, nationality and
vocation, and some men would be included in more
than one species; for example, artists who are French-
men, and mechanics who are Germans.

RULE IV: *In a series of divisions and subdivi-
sions each genus or class should be divided into its
proximate members or species.*

The **proximate members** of a genus are the mem-
bers or species which together immediately make up
the genus. **Remote members** of a genus are mem-
bers which together immediately make up any of
the species into which the genus has been divided.
Thus, if plane figure is divided into curvilinear and
rectilinear, it is divided into its *proximate* members.
Rectilinear figure is in turn divided into its proximate
members, if it is divided into triangle, quadrilateral,
and polygon. Triangle, quadrilateral, and polygon are
therefore *remote* members of plane figure; and hence,
Rule 4 would be violated, if they were set down as the
dividing members of plane figure. Again, the viola-
tion of this rule would occur in an aggravated form,
if, plane figure were divided into curvilinear figure,
polygon, and isosceles triangle.

CHAPTER XVII

DEFINITION

111. The object in view in logical division is the arrangement of our ideas on a subject in orderly sequence. The purpose of definition is the acquisition of clear or distinct ideas on the subject under discussion (cf. 6). The framing of good definitions is by no means easy. On most subjects it demands a power of analysis which comes only after long mental training. And even with the requisite preparation of mind a man will often have to devote himself to a long and laborious study of a subject before he hits upon a definition which will be satisfactory.

The word "definition" is applied to the process of analysis and comparison which results in a clear or distinct idea of the subject under consideration, and also to the verbal expression of this idea. Viewed as a process, definition is the process of differentiating briefly the thing denoted by a given term from everything else. In Logic we are concerned mainly with the verbal expression of the clear or distinct idea which is acquired by this process. This must, accordingly, be kept in mind in the course of the explanation which follows.

Definition is the explicit verbal expression of a clear or distinct idea by which the thing denoted by a

given term is briefly differentiated from everything else.

Two things should be noted here.   First, in a definition the thing denoted by the term is *briefly* differentiated.   If details are introduced which are not required to differentiate the thing, or if the differentiation involves a long explanation, the result is not called a definition.   Secondly, it is a *term,* not an idea or a thing, which is defined.   Commonly the definition stands in the predicate position of a categorical proposition, and the term defined, in the subject position.   In such a proposition the subject expresses a vague or confused idea, and the predicate, a clear or distinct idea of the same object.   When a definition is given, then a thing is differentiated, a clear or distinct idea of the thing is explicitly expressed, and a term is defined.   However, we do sometimes speak loosely of defining a thing; but this really means defining the term which stands for the thing.   After the term has been defined, the term, which before conveyed but a vague or confused idea, is henceforth, to the person who knows the definition, the expression of a clear or distinct idea, and he has no further need to employ the longer and more explicit expression.

NOMINAL AND REAL DEFINITION

112. A **nominal definition** is a definition which tells what *thing* is denoted by a term.   Its function is to secure agreement as to the *thing* which is under discussion.   Nominal definition is intended to convey a *clear* idea.   It may also convey a distinct idea to a

person who knows the definition of the words em-
ployed in the nominal definition; but that is acci-
dental.

A **real definition** is a definition which tells what the
thing *is* which is denoted by a term.   Its function is
to *explain* the thing which is under discussion.   Real
definition conveys a *distinct* idea of the thing denoted
by the term.

Nominal definition is used when the object denoted
by the term is not clear to the persons addressed,
either because the term is not familiar to them, or be-
cause it is ambiguous, or because it is employed in a
special sense.

Works on Logic commonly enumerate four kinds
of nominal definition, as follows: 1. A definition
which gives the etymology of a word; e.g. "A martyr
is a witness;" "An Angel is a messenger."   2. A defini-
tion which translates a foreign word; e.g. "Dux is a
commander."   3. A definition which substitutes a
synonym which is understood for a word which is not
understood; e.g. "A prestidigitator is a juggler."   4. A
definition which assigns to a term a special sense in
which it is to be used in the course of a particular
discussion.   This fourth kind of nominal definition is
called **special, conventional,** or **technical.**

VARIOUS KINDS OF REAL DEFINITION

113. It is only a *general* or an *abstract* term that can
be defined by a real definition.   When a determinate
individual, that is, an object denoted by a *singular*
term, is differentiated from all other objects, the result
is a description or, at best, a nominal definition.

An **essential definition** is a definition which gives the proximate genus and specific difference of an object according to some classification. Thus, the essential definition of "man" is either "rational animal" or "sentient rational being," according to the classification (cf. 103). The essential definition of "orator" is "a man gifted with the power of persuasion in public speech." If we were to say "Man is a rational organism," we should give a real, but not an essential, definition of "man," because "organism" is not the *proximate* genus of man (cf. 104, 103).

Ordinarily, it is only when the definition states explicitly or implicitly those attributes which are by common consent the *most fundamental* in the object, that it is called an essential definition. Such a definition explains the *nature* of the object (cf. 102, N.B.). When other attributes are indicated, the definition usually belongs to one of the classes described below. It is extremely difficult to determine what attributes are the most fundamental in the objects that present themselves to us; for this reason, we have for the most part to content ourselves with something short of an essential definition. The nature of many subjects of scientific study has thus far escaped detection by the most prolonged and minute analysis. Hence it is that many of the definitions in scientific treatises are little more than nominal definitions.

A **genetic definition** is a definition which indicates the process by which the thing denoted by the term is produced; e.g. "A circle is a figure formed by a point moving in a plane at a constant distance from another point in the same plane."

N. B.—It is to be noticed that, in genetic definition and in the others that follow, some attribute is generally selected to serve as a genus and that the element peculiar to each class of definition is assigned as a specific difference.

A **distinctive definition** is a definition which indicates a property of the thing denoted by the term (cf. 102, N.B.); e.g. "Man is an animal capable of laughing."

A **descriptive definition** is a definition which indicates a group of attributes which are other than properties and the most fundamental attributes and which are found together only in the thing denoted by the term; e.g. "A tiger is a large feline mammal with vertical, wavy stripes of black and tawny color on the body."

A physical **definition** is a definition which indicates the physical parts which compose the thing denoted by the term; e.g. "Man is a being composed of a rational soul and an organized body."

A **causal definition** is a definition which indicates an extrinsic cause of the thing denoted by the term; e.g. "A honeycomb is a waxen structure consisting of hexagonal cells ranged side by side, made by bees" *(efficient* cause); "A saddle is a contrivance fastened to the back of an animal to support a rider" *(final* cause); "A statue is a plastic work made after the likeness of a human or animal figure" *(exemplar* cause).

LIMITS OF REAL DEFINITION

114. There are three main classes of terms which do not admit of real definition, viz. proper names,

terms which denote the objects of our simplest ideas, and terms which denote the objects of our elementary experience.

(1) A proper name cannot be defined by a real definition, because the object denoted by it, being a determinate individual, has such a multitude and variety of aspects, that to attempt to include in the compass of a definition a sufficient number to differentiate the individual would only lead to confusion.

(2) The reason why the second class of terms cannot receive a real definition is plain. Real definition involves a process of analysis, in the course of which a complex object of thought is resolved into simpler aspects. Hence, if the object is not complex, as is the case with the objects of our simplest ideas, there is no room for analysis, and consequently, no possibility of a real definition. For this reason, we are unable to construct a real definition of the ten Categories.

(3) The objects of our elementary experience cannot be explained by a real definition, because they are better known to us than anything else; and besides, any reference to them is intelligible only in the light of previous experience. Hence, there is no expedient we might resort to which could possibly convey an idea of them to a person who had had no experience of them. Thus, the terms which denote the objects of our internal and external sensations are incapable of real definition. Such terms are, for instance, "pain," "pleasure," "fragrance," "greenness," "sweetness," "loudness."

N. B.—Most of the terms we have just referred to can, of course, be defined by a *nominal* definition.

RULES OF REAL DEFINITION

115. RULE I: *The definition must be co-extensive in application with the term to be defined.*

An alternative statement of the rule is this: *The definition must be simply convertible with the term to be defined.* This rule follows from the very nature of definition, which is to differentiate the thing denoted by the term from everything else. This differentiation is not effected, when what is offered as a definition has a wider or a narrower application than the term to be defined. "Man is a two-legged animal:"—the predicate of this proposition is not a definition of "man;" it has a *wider* application than "man;" for it does not differentiate man from bird. "A proposition is the verbal expression of a judgment:"—here the predicate is not a definition of "proposition;" it has a *narrower* application than "proposition;" for it does not differentiate the *hypothetical* proposition from the argument or the term.

RULE II: *The definition must convey a more distinct idea of the thing than the term to be defined.*

This rule follows from the nature of *real* definition, the function of which is to *explain* the thing denoted by the term. A real definition is to the term defined as a distinct idea is to a confused idea. Hence, this rule forbids especially the use of ambiguous or figurative words or phrases in a definition. The following examples are violations of this rule: "The lion is the king of beasts;" "Bread is the staff of life;" "Life is the definite combination of heterogeneous changes, both simultaneous and successive, in correspondence with external co-existences and sequences."

It is scarcely necessary to say that Rule 2 means that the definition must convey a more distinct idea to the persons who know the meaning and force of the words employed in it. A sentence in English does not cease to be clear because it is unintelligible to a man who is unacquainted with the English language.

A violation of Rule 2 is called an attempt to explain **obscurum per obscurius** or **ignotum per ignotius**.

RULE III: *The definition must not contain the term to be defined nor an equivalent word.*

When this rule is not observed, one and the same idea underlies the term to be defined and the definition, and consequently, the definition does not convey a more distinct idea than the term to be defined. This rule is violated in the following examples: "Man is an animal endowed with human nature;" "Justice is the way of acting justly;" "Life is the sum of vital processes."

The violation of this rule in a single proposition is called defining **idem per idem**.

It is also accounted a violation of this rule when a term, after being defined, is used to define one of the words in its own definition. Such a violation of the rule is called a **Circulus in Definiendo**. The following is an example of it: "A day is a period of time consisting of twenty-four hours," "An hour is the twenty-fourth part of a day." This "circle" would be avoided, if "day" were defined as "the period of time from sunrise to sunrise." It is to be observed, however, that when a pair of *relative* terms, like "parent"—"child," are to be defined, each must appear in the definition of the other; for neither is intelligible without the

other. Hence, this would not be called a violation of Rule 3. In many cases the definition of a relative term will be a nominal, not a real, definition.

RULE IV: *A term which admits of a positive definition must not be defined negatively.*

This rule also follows from the nature of real definition. Real definition tells us what the thing *is* which is denoted by a term, not what the thing is *not*. The following examples violate this rule: "A body is a thing that is not spirit;" "Wisdom is the avoidance of folly."

A term which is negative, not in appearance alone, but as expressing a negative idea, cannot, of course, be defined positively. For example, "darkness" can only be defined negatively, that is, by saying that it is the absence of light. Again, when the idea expressed by a term has been acquired by a process of elimination, the term can be defined only by reference to what has been eliminated, that is, negatively, though what it denotes is something positive. For example, "A point is a thing that has position, but not dimension."

CHAPTER XVIII

FALLACIES

116. The study of fallacies in Logic is not intended to provide the mind with an absolute safeguard against error. That would be an impossible task, considering the limitations of the human intellect and the multiplicity, variety, and abstruseness of the subjects which present themselves for investigation. Logic can, however, put us on our guard against the more common pitfalls into which the multitude of men are betrayed. A man who, in addition to an intimate acquaintance with the principles of Logic, possesses a ready familiarity with these sources of error will have a distinct advantage over the ordinary man, and that in three principal ways. First, he will be in less danger of employing a fallacious argument for the support of his own position. Secondly, he will be less likely to fall a victim to the fallacious arguments of others. Thirdly, he will be able to expose the weakness of an opponent's argument with greater clearness and effectiveness.

It will be well to discriminate between the four following terms:

A **fallacy** is an inconclusive argument which appears to be conclusive.

A **paralogism** is an argument which openly violates a rule of the syllogism.

171

A **sophism** is a fallacy constructed for the purpose of deceiving.

A **paradox** is a statement at variance with received opinion.

It is to be observed that ordinarily an inconclusive argument appears to be conclusive only when it is so long and complicated that it is difficult for the mind to take a connected view of all its successive steps. The obscurity which attends upon protracted and involved argumentation is the favorite lurking-place of fallacy. But let a fallacious argument be exhibited in skeleton form, and it commonly loses all its plausibility, and thus ceases to be a fallacy. The invalid arguments we are about to use as illustrations would hardly impose on anyone. But they will serve at least to bring out what the fallacious argument amounts to, when it is stripped of everything that is extraneous to it.

In the conduct of a discussion three points should be insisted on, as having an important bearing upon the legitimate issue of the argument. First, all elliptical and figurative expressions employed in the statement of the reasoning process should be explained in their full and literal sense. Secondly, the meaning in which the prominent terms are to be used should be made clear from the outset and this meaning rigidly adhered to throughout the discussion. Thirdly, the precise point to be proved should be set down explicitly in the beginning and kept steadily in view during the progress of the argument.

117. In explaining the fallacies we shall follow the order adopted by Aristotle in the list which he drew

up for the solution of the fallacious arguments of the
ancient Sophists. Aristotle divided fallacies into two
groups, namely, Fallacies *in dictione* and Fallacies *extra
dictionem*. A fallacy **in dictione** is a fallacy arising
from the mode of expression. A fallacy **extra dic-
tionem** is a fallacy arising from something distinct
from the mode of expression.

Aristotle's list of fallacies is as follows:

In dictione—
1. Equivocation
2. Amphiboly
3. Composition
4. Division
5. Accent
6. Verbal Form, or *Figura dictionis*

Extra dictionem—
1. Accident, or Moral Universal
2. *Secundum quid,* or Special Case
3. *Ignoratio elenchi,* or Evading the Question
4. *Petitio principii,* or Begging the Question
5. *Non causa pro causa,* or Fabricated Absurdity
6. Consequent, or *Non sequitur*
7. Complex or Insinuating Question

FALLACIES in dictione

118. **Equivocation** is the fallacy of employing the
same word or phrase in different senses in the same
argument.

When the fallacy of Equivocation is perpetrated in a
categorical syllogism, it is usually the middle term

that is ambiguous. For example—

> . All criminal actions ought to be punished
>   by law,
> Prosecution for theft is a criminal action,

Therefore Prosecution for theft ought to be pun-
ished by law.

In the major premise "criminal action" is the same
as "crime;" in the minor it means a kind of legal
process.

> That which is rare is dear,
> Cheap horses in Paris are rare,

Therefore Cheap horses in Paris are dear.

In the major premise "rare" means "precious;" in
the minor it means "seldom met with."

119. **Amphiboly** is the fallacy arising from ambigu-
ity in the structure of a sentence.

The prophecies of the oracle of Delphi were gen-
erally couched in this ambiguous form, so as to admit
of a true interpretation, whatever might be the issue
of the enterprise to which they referred. Thus, before
his invasion of Italy the following prophecy was made
to Pyrrhus: "Aio te, Æacida, Romanos vincere
posse," "I declare, son of Æacus, that you the Romans
can conquer." In Shakespeare's *Henry VI* the witch
prophesies that "the duke yet lives that Henry shall
depose." The following are additional examples of
the fallacy: "Twice two and three" may be seven or
ten, according as the multiplication is performed be-
fore or after the three is added. "Wolsey left at his
death many buildings which he had commenced in an
unfinished state." "Wanted: A colored man for cook-
ing."

120. **Composition** is the fallacy of concluding that what is true of certain elements when taken separately is also true of them when taken together.

We should commit this fallacy, if we asserted that the unanimous verdict of a jury was liable to be wrong, because each individual juryman was incapable of forming a calm and balanced judgment upon the testimony; that is, we should overlook the fact that twelve heads are better than one. Again, this fallacy would be committed by a man who should pronounce that the cumulative force of various independent testimonies in favor of a fact failed to constitute a proof of the fact, because a single witness was liable to be mistaken. As the stones of an arch support each other and are thus able to sustain the building, so the independent testimony of each witness strengthens that of the others, and is itself strengthened in return, so that the combined force of all is able to do what none of them could do, if taken in isolation. Each testimony is to be viewed in relation to the others; for they all form a *connected* and *converging* body of proof. He commits the fallacy of Composition who likens the cumulative proof to a chain; for the links in a chain do not strengthen each other, and one defective link renders the whole chain as weak as itself.

121. **Division** is the opposite of Composition; it is the fallacy of concluding that what is true of certain elements when taken together is also true of them when taken separately.

A man would fall into this fallacy, if he argued that he was not bound to give aid to this poor man, nor to

a second poor man, nor to a third, and so on, because he was not bound to aid them all.

N. B.—The fallacy of Composition and the fallacy of Division are in reality fallacies *extra dictionem.* The following fallacy of Composition and Division is a fallacy *in dictione.*

122. The name of **Composition and Division** is given to the fallacy when, in the compass of a single syllogism, several elements are treated as united after having been treated as separate, or *vice versa,* and also when there is a confusion of universal (distributive) with collective or vague supposition (cf. 18).

The following are instances of this fallacy:

> That which happens almost every day is probable,
> Improbable events happen almost every day,

Therefore Improbable events are probable.

In the minor premise the term "improbable events" is used vaguely; in the conclusion it is used universally; in the minor premise it means "some improbable event or other;" in the conclusion it means "every improbable event." Moreover, the words "probable" and "improbable" are used as though they denoted an absolute attribute which inhered in the thing or event itself; whereas, in reality, they denote the attitude of our minds towards the thing or event. "Probable" means "to be expected," and "improbable" means "not to be expected" or "unexpected." It is also to be observed that in the foregoing syllogism "improbable

event" does not mean "event which *is* unexpected," but "event which *has been* unexpected." The syllogism may therefore be corrected as follows:

> That which happens almost every day is to be expected,
>
> Some event or other which has been unexpected happens almost every day,
>
> Therefore Some event or other which has been unexpected is to be expected.

The conclusion, stated in other words, would run, "We must expect to be surprised from time to time." This example may be explained in another way. We may interpret the major premise as meaning "That which repeats itself almost every day is probable." Then it is false to say in the minor premise, "Improbable events repeat themselves almost every day."

> He who necessarily goes or stays is not free,
>
> You necessarily go or stay,
>
> Therefore You are not free.

In the major premise "necessarily" modifies "goes" and "stays" separately, so that the meaning is "He who necessarily goes or necessarily stays is not free;" in the minor premise "necessarily" modifies "go or stay" taken together.

> Two and three are two numbers,
>
> Five is two and three,
>
> Therefore Five is two numbers.

In the major premise "two" and "three" are taken separately; in the minor they are taken together.

> All the angles of a triangle are equal to two
>   right angles,
> All the angles of a triangle are less than
>   two right angles,
> Therefore Something less than two right angles is
>   equal to two right angles.

In the major premise "angles of a triangle" is used collectively; in the minor it is used distributively.

> All the angles of a triangle are equal to
>   two right angles,
> The angle A is an angle of a triangle,
> Therefore The angle A is equal to two right angles.

In the major premise "angles of a triangle" is an elliptical expression for "angles of a triangle taken together;" in the minor there is no such ellipsis.

> The blind cannot see,
> But we read in Scripture that the blind see,
> Therefore We read in Scripture that the blind do
>   what they cannot do.

The major premise is true *in sensu composito;* the minor, *in sensu diviso.* In the major premise "the blind" means "those who *are* blind;" in the minor it means "those who *were* blind" (cf. 18, ad fin.).

123. **Accent** is the fallacy of stressing a wrong syllable or of unduly emphasizing some word or phrase in the sentence of another person.

This fallacy is committed in quotations, when a syllable, word, or phrase is stressed which was not stressed by the author of the passage. Thus, someone says: "The King was incensed at the celebration,"

meaning that the King was angry; and a speaker, in quoting this sentence, puts the accent on the first syllable of "incensed." When italics or punctuation marks are introduced into a quotation or omitted from it, the person who is quoting should make it clear that he is responsible for the insertion or the omission. A man would commit the fallacy of accent, if he omitted the comma after the word "too" in the following sentence: "On came the cavalry, too, eager for the battle."

Under this head may be classed the fallacy of **Special Pleading,** which consists in dwelling upon the points which are favorable to the cause which has been espoused and slurring over those which are disadvantageous.

The form in which the fallacy of **Quibbling** usually exhibits itself is such as almost to warrant us in classifying it under Accent. The fallacy of Quibbling is commonly the product of a small and petty spirit in argumentation, which seizes upon points of minor importance and emphasizes them as though they were the essential factors in the settlement of the question under discussion. Again, the quibbler will distort the words of another man by attaching to them a sense which the man did not intend them to convey, and which an intelligent and well-informed person would see that they could not have been meant to convey.

124. **Verbal Form** or **Figura dictionis** is the fallacy of ascribing a similar force to certain words or phrases because of similarity of structure.

This fallacy would be committed by a person who should conclude that, because the two first letters in

"inactive" and "insolvent" have a negative force, they have the same force in "intelligent" and "instinctive." There are persons who think that the termination of the word "Catholicism" has the force of the word "schism." J. S. Mill was guilty of this fallacy when he argued that, because what is seen is *visible* and what is heard is *audible,* therefore what is desired is *desirable.* But "visible" and "audible" mean what *can* be seen and what *can* be heard respectively; whereas "desirable" in the conclusion which Mill inferred does not mean what *can* be desired, but what *ought* to be desired.

> He who has least to read reads least,
> He who reads most has least to read,

Therefore He who reads most reads least.

In the major premise "reads least" means *"will read* least;" in the minor "He who reads most" means "He who *has read* most." Accordingly, the conclusion should be, "He who has read most will read least."

FALLACIES extra dictionem

125. **Accident** or **Moral Universal** is the fallacy of concluding that what is asserted of a subject apart from a given attribute, state, or circumstance may be asserted of the subject or its inferiors with direct reference to that attribute, state, or circumstance (cf. 3, 8).

> The things bought from a fruiterer are eaten,
> Unpeeled oranges are bought from a fruiterer,

Therefore Unpeeled oranges are eaten.

In the major premise "eaten" is predicated of "things bought from a fruiterer" apart from any particular state; in the conclusion it is asserted of certain inferiors of "things bought from a fruiterer" with direct reference to a particular state.

> A citizen has a legal right to inculcate his private doctrines,
>
> Teachers in schools are citizens,
>
> Therefore Teachers in schools have a legal right to inculcate their private doctrines.
>
> A man has a right to walk in the city,
>
> A murderer is a man,
>
> Therefore A murderer has a right to walk in the city.

It will be found that the fallacy of Accident consists for the most part in treating as *absolutely* universal what is only *morally* universal (cf. 23).

> He who says you are a man is right,
>
> He who says you are a Hottentot says you are a man,
>
> Therefore He who says you are a Hottentot is right.

The major premise is only *morally* universal, and is true of those persons alone who say that you are a man and neither assert nor imply anything else.

> Every rule has an exception,
>
> This major premise is a rule,
>
> Therefore This major premise has an exception.

The major premise is not *absolutely* universal.

> He who kills a fellowman is guilty of grave crime,
>
> Many a soldier in battle kills a fellowman,
>
> Therefore Many a soldier in battle is guilty of grave crime.

126. Secundum quid or **Special Case** is the opposite of Accident; it is the fallacy of concluding that what is asserted of a subject or an inferior of a subject with direct reference to a given attribute, state, or circumstance may be asserted of the subject apart from that attribute, state, or circumstance.

This fallacy consists in arguing from a special case to a universal rule. By an extension of the fallacy it is made to cover the fallacy of arguing from one special case to another when the cases are not parallel; the fallacy is then called **False Analogy.**

The following are instances of the fallacy in its first form: Someone argues that we are never obliged to tell the truth, because it is right to withhold it from an enemy in time of war; or that wine is a poison, because, when taken in excessive quantities, it is injurious to the health; or that it is wrong to give alms to the poor, because the bestowal of money on the indolent encourages them in their slothfulness.

It would be an example of the fallacy under the second form, if a person were to argue that, since excessive drinking ruins many families, therefore the person who opposes absolute prohibition helps to ruin many families.

127. Ignoratio elenchi or **Evading the Question** is the fallacy of refuting or conceding something as though it had been maintained, whereas it had not been maintained.

Ignoratio elenchi means literally ignorance of the contradictory of an opponent's assertion. When it occurs, it is usually in a refutation. Unless the *con-tradictory* of an opponent's statement is established,

he has not been refuted. The fallacy goes by various names, such as "arguing beside the point," "missing the point at issue," "irrelevant conclusion." The statement that the study of the classics and the sciences affords an unequalled mental discipline is not refuted by proving that that study produces no immediate results in dollars and cents. Again, the defendant's protest that he did not commit the abominable crime of murder is not refuted by proving that murder is an abominable crime. It is not an uncommon thing for the champion of some scientific theory to impute to his opponents a contempt for science, when the real point at issue is whether his theory is true.

An instance of *ignoratio elenchi* occurs in the controversy between Newman and Kingsley. Kingsley charged Newman with saying that truth for its own sake need not, and on the whole ought not to be, a virtue with the Roman Clergy. Newman protested that he had never made such an assertion, and challenged Kingsley to prove his charge. Kingsley did not attempt to prove it and consented to withdraw it; but instead of saying that Newman had not made the statement in question, he said that Newman had assured him that the statement was not intended to convey the meaning which Kingsley had put upon it.

According to the direction taken by the argument, the fallacy of *ignoratio elenchi* is divided into various sub-classes.

When the argument meets the issue squarely, it is, of course, not a fallacy, and it is then called an **argumentum ad rem.**

The ·fallacy of *ignoratio elenchi* is not unfrequently

resorted to by those who oppose the introduction of a measure of a practical nature. There is no such thing as an ideally perfect practical measure, that is, a measure which has no difficulties or inconveniences attendant upon its enforcement. When, therefore, a measure of this character is proposed, it is no disproof of its expediency to show that it has certain disadvantages connected with it. What has to be proved is that these disadvantages *outweigh* the disadvantages experienced under the existing state of things. When the fallacy assumes this form, it is called the **Fallacy of Objections.**

The **argumentum ad baculum** is an appeal to physical force; as when a strong nation, by the threat of invasion, extorts a concession from a weaker nation, or when a disputant, by loud and continuous talking and by pounding on the table, attempts to stifle all opposition to his contentions.

The **argumentum ad populum,** or the "appeal to the gallery," consists in appealing to popular prejudices and passions on a matter which should be argued out on its own merits. This fallacy is a very common trick with demagogues and political campaigners.

The **argumentum ad ignorantiam** is the fallacy of arguing that a suggested fact or theory cannot be proved to be impossible, when it should be proved positively to be *true* or *probable*. A disputant resorts to this fallacy when he attempts to win over his hearers to the acceptance of his doctrine by pointing out that no one can prove the doctrine to be absurd, whereas he ought to advance positive reasons for its truth. Under this head belongs the trick of shifting the burden of

proof from one's own shoulders, where it properly belongs, to those of an opponent. It should be observed, however, that when our avowed purpose is not to establish the truth of a given proposition, but to clear it of the charge of absurdity, it is perfectly legitimate to insist that the opponents of the proposition . cannot prove it to be absurd.

It is also considered an *argumentum ad ignorantiam,* when a speaker or writer, with a view of getting a certain measure or proposition adopted, puts forth a number of assertions in the expectation that they will be accepted by an ignorant and gullible public. This is the method commonly adopted by the unscrupulous agitator in his attacks upon a person or an institution. He acts upon the principle, "Throw dirt enough, and some will stick."

The **argumentum ad verecundiam** is an attempt to settle a question by appealing to the reverence which is felt for a great name or a long-established custom, when it ought to be decided on its intrinsic merits. The authority which attaches to immemorial usage and to the pronouncements of men eminent in their own sphere of study or activity should never, of course, be lightly set aside, and the burden of proof always rests with those who come in conflict with it. Authority is "in possession," and has a prescriptive claim upon our allegiance. But when the evidence in favor of a given proposition is clear and unmistakable, it is a fallacy to say, "But this great man is against you" or "The practice of years lies in a different direction."

Sometimes the present fallacy is encountered in a

peculiarly aggravated and mischievous form. A man who is distinguished in a particular field is occasionally asked to give his opinion concerning some subject upon which he is unfitted, whether by study or habit of life, to pronounce a trustworthy judgment. The general of an army justly resents the appeal to a professor of physical science on a question of military tactics. But it is especially in connection with subjects bearing upon education and religion that this fallacy is committed. If the venerable maxim, "Sutor ne supra crepidam," were more generally kept in mind by men of reputation, they would not be betrayed so frequently into shallow, offhand judgments upon matters beyond their province. The solemn flippancies which so often fall from the lips of these men are a clear proof that a distinguished man is not necessarily a wise man.

The **argumentum ad misericordiam** is an attempt to arouse pity and sympathy for an accused person in order to divert attention from the question of his guilt. Nowadays it is a very ordinary occurrence for misguided, sentimental busybodies to besiege the courts with petitions in behalf of the most cold-blooded villains.

The **argumentum ad hominem** is an attempt to discredit or recommend a proposition by appealing to the character or the present or past professions of the person against whom the argument is directed. The attorney for the defence in a certain lawsuit is said to have written on his brief, "No case; abuse the plaintiff's attorney." Abuse, sarcasm, recriminations, charges of inconsistency, and the like, are the com-

mon tactics of men who are more anxious to triumph
over an opponent than to establish the truth. Thus,
it not unfrequently happens that a legislator is
accused of acting against his former professions in
proposing a certain measure, when the real point at
issue is whether the measure would benefit the
country.

The *argumentum ad hominem* is not always irrele-
vant: it has its legitimate uses. It is a perfectly lawful
expedient, when it is employed to silence a captious
opponent or to open the eyes of those who are pre-
vented by ignorance, inadvertence, or prejudice from
appreciating the true force of an argument or the
weakness of their own position. It was thus that our
Lord put His adversaries to shame when, in reply to
the protest of the Ruler of the Synagogue against
His healing on the Sabbath, He asked, "Doth not
every one of you, on the Sabbath day, loose his ox or
his ass from the manger, and lead them to water?"
Again, a lawyer may rightfully impugn the testimony
of certain witnesses on the ground that their evil
character renders them unworthy of credence.

N. B.—Owing to the ease with which the real point
in dispute may be obscured, it is a wise policy for a
disputant to preface his discussion of a question by
disengaging it from false issues and forestalling mis-
interpretations.

128. **Petitio principii or Begging the Question** is the
fallacy of assuming in the proof what has to be proved.

Before any argument has been advanced to prove
a proposition which requires proof, the proposition is

called a *question:* hence the appropriateness of the name, "Begging the Question."

The fallacy of Begging the Question has two principal forms. In the first, the conclusion is assumed under cover of synonymous words or expressions; in the second, it is assumed in the guise of a universal premise which cannot be accepted unless the conclusion is already *known* to be true. The fallacy in the first form is perhaps more easily committed in English than in any other language, owing to the various sources from which English words are derived. Thus, two sets of words which express absolutely the same meaning may be made to look like two different propositions. The following examples may serve as illustrations: "To allow every man an unbounded freedom of speech must always be, on the whole, advantageous to the State; for it is highly conducive to the interests of the Community that each individual should enjoy a liberty perfectly unlimited of expressing his sentiments." "The young will abstain from vice if the physical consequences of vice are pointed out to them, because a knowledge of those consequences cannot fail to deter them from committing vicious acts."

A man would be guilty of the fallacy in its second form, if he were to argue that the bestowal of the suffrage upon women will benefit the country, because the country must be benefited by an arrangement which apportions to men and women the same duties and responsibilities.

This fallacy is frequently committed by the partisans of a scientific or philosophic theory. They lay

down their theory dogmatically as absolutely true, and obstinately refuse to listen to any evidence which might tend in an opposite direction. For them a fact ceases to be a fact, when it tells against their theory.

When a proposition is advanced to prove a given conclusion, and then the conclusion itself employed to prove the first proposition, the fallacy is called a **Vicious Circle** or **Arguing in a Circle (Circulus in probando).** For example, a disputant asserts that an undergraduate who studies the classics is wasting his time. When asked for his reasons, the disputant replies, because he is not fitting himself for the serious duties of life. When, again, he is asked why the undergraduate is not fitting himself for those duties, the answer is, because he is wasting his time on the classics.

In this connection should be mentioned the **Question-begging Epithet.** This is a commendatory or disparaging epithet which is attached to a person or an object before it has been proved to be deserved. We constantly find men applying to a doctrine or cause a name which, by its implication, begs the whole question as to the truth, achievements, or prospective benefits of the doctrine or cause. To this category belong many party names and political catchwords, which are invented for the purpose of securing votes; for example, "the full dinner-pail," "progress and prosperity," "the party of the people." Such, again, is the facile use which certain writers make of a name like "the savage Cave Man," as though it were a well-established fact that there had been any savage cave men. The following words of Mr. G. K.

Chesterton are in point here: "Don't say (O don't say) that Primitive Man knocked down a woman with a club and carried her away. Why on earth should he? Does the male sparrow knock down the female sparrow with a twig? Does the male giraffe knock down the female giraffe with a palm tree? . . . Utterly and absolutely abolish all such bosh—and we may yet begin to discuss these public questions properly."

The disparaging epithet is probably of more frequent occurrence than the epithet which favors one's own doctrine. An evil or derisive appellation, unjustly fastened upon a man, is often the cause of widespread prejudice against him. The words "clever" and "ingenious" may have a good meaning; but in many cases they are applied to a person with the added implication that he is lacking in depth and truth. To call a writer clever or ingenious or brilliant is a short and easy method of prejudicing him in the minds of others and of escaping the irksome task of weighing and answering his arguments.

129. **Non causa pro causa** or **Fabricated Absurdity** is the fallacy of fastening upon an opponent's statement an absurdity which does not follow from it, but from something else which has been substituted for it. In the expression, "non causa pro causa," the word "causa" has the same meaning as the English word "reason."

This fallacy is a pretended application of the indirect disproof, that is, of the *reductio ad absurdum* (cf. 90). The *reductio ad absurdum*, in its legitimate use, disproves a proposition by showing that its acceptance would place a man in an absurd position.

But the person who resorts to the fallacy of *non causa pro causa* merely shows that an absurdity attaches to a proposition which he falsely ascribes to his opponent. Let us suppose, for example, that a legislator protests against treating crime as a disease. His opponent proceeds to argue as follows: "Your position involves you in an absurdity; for if we are never to take into account the physical condition of a person who perpetrates a deed of violence or passion, then the wild acts of a lunatic or of a man in a delirium ought to be visited with penal servitude." The absurdity which is here imputed to the legislator does not follow from his statement, but from a proposition which has been unfairly fathered upon him, namely, that we are never to take into account the physical condition of a person who perpetrates a deed of violence or passion.

Under the head of *non causa pro causa* is usually classed the fallacy of **False Cause,** or **post hoc, ergo propter hoc.** This is the fallacy of setting down as the cause of an event something which merely precedes or accompanies it. A man indulges freely in wine at his club and wakes up the next morning with a splitting headache, and he attributes his indisposition to the intense mental application with which he has been prosecuting his business. The clouds are dispersed, and this is ascribed to the rising of the full moon. A man is irritable in the morning, and his relatives account for it by the fact that he got up on the left side of the bed.

130. **Consequent** or **Non sequitur** is the fallacy of drawing a conclusion from premises with which it has no logical connection.

With one or two exceptions, all the fallacies which were noticed in connection with Eduction and the Hypothetical and the Categorical Syllogism belong to the class of *non sequitur* arguments. The name of *Consequent* is especially appropriate to the fallacies of the Mixed Hypothetical Syllogism, viz. inferring the truth of the antecedent from the truth of the consequent, and inferring the falsity of the consequent from the falsity of the antecedent.

It may be useful to set down a few examples of this fallacy which differ somewhat from those previously mentioned.

> All propositions are true or false,
> All propositions are not true,
> Therefore All propositions are false.

In a negative proposition the sign "all" has the same force as "some" (cf. 25). Hence the syllogism should read:

> All propositions are true or false,
> Some propositions are not true,
> Therefore Some propositions are false.

"A bottle half-empty is equal to a bottle half-full: multiply both sides of the equation by two, and the result is: An empty bottle is equal to a full bottle." *Answer:* If both sides of the equation are multiplied by two, the result will be: "Two bottles half-empty are equal to two bottles half-full."

> I am a man,
> You are not what I am,
> Therefore You are not a man.

The minor premise is not true, unless it means "You are not everything that I am." If it means "You are nothing that I am," it is false. Besides, there are four terms in the argument. The syllogism should be worded as follows:

> I am a man,
> You are not I,
> Therefore You are not a man.

The argument now violates the rule of the First Figure which requires the minor premise to be affirmative (cf. 71).

> Either it is raining or it is not raining,
> But it is not raining,
> Therefore It is raining.

This argument violates the rule of the Disjunctive Syllogism which prescribes that, when one alternant is sublated in the minor premise, the remainder of the disjunctive premise shall be *posited* in the conclusion (cf. 56, 59). The alternants in the foregoing argument are "it is raining" and "it is not raining." Since the alternant, "it is raining," was sublated in the minor premise, the remaining alternant, "it is not raining," should be posited in the conclusion. The syllogism, correctly stated, will run as follows:

> Either it is raining or it is not raining,
> But it is not raining,
> Therefore It is not raining.

> Either the affirmative or the negative side
>   is sure to win,
> But the affirmative side is not sure to win,
> Therefore The negative side is sure to win.

The major premise in this argument is not a disjunctive proposition. In this premise we have an instance of confused or vague supposition (cf. 18).

"When one raindrop is joined to another, they form into one body, and there are no longer two raindrops. Therefore, one plus one is not necessarily equal to two." *Answer:* When a person says that one plus one is equal to two, it is obvious that he means that where there is one plus one, there are two; he certainly does not mean that where there is *not* one plus one, there are two. When one raindrop is joined to another, there is *not* one plus one. One plus one can cease to be equal to two only by ceasing to be one plus one.

> Either all will pass or some will fail,
> But some will not fail,
Therefore All will pass.

The alternant "some will fail" is not sublated in the minor premise. To sublate is to set down as false. In order to sublate this alternant, we shall have to assert its contradictory, viz. "None will fail."

> That which I see in the distance may be
> a cow,
> That which I see in the distance is a wagon,
Therefore A wagon may be a cow.

In the first place, in accordance with the third rule of the Third Figure, the conclusion should at the most be "Some wagon may be a cow" (cf. 73). In the second place, the time-reference in the major premise is different from the time-reference in the minor. And besides, there is a play on the words "may be." In the major premise "That which I see in the distance"

is viewed with reference to the time when it makes
only a vague impression on my senses; so that the
meaning of the premise is "That which I see in the
distance is a thing that I cannot distinguish from a
cow." In the minor premise "That which I see in the
distance" is viewed with reference to a subsequent
time when I see it to be what it really is. Hence, the
major and minor premises cannot both be uttered by
the same person with reference to the same moment
of his experience. The major premise is true of one
moment of his experience; but it is false with refer-
ence to the moment in which his senses testify to the
truth of the minor premise. Suppose, however, that
the minor premise is an item of information which is
communicated to him by others at the moment of the
vague impression recorded in the major premise; then
the syllogism will have to be expressed as follows:

> That which I see in the distance is a thing
> that I cannot distinguish from a cow,
> But that which I see in the distance is re-
> ported to be a wagon,

Therefore Something reported to be a wagon is a
thing that I cannot distinguish from a
cow.

> No mere animal has a language,
> A deaf-mute is no mere animal,

Therefore A deaf-mute has a language.

The word "no" in this argument may be interpreted
in two ways: first, as forming part of the middle term,
so that the term is "no mere animal" or "being that
is not a mere animal;" secondly, as a particle which

makes both premises negative. On the first interpretation the argument will read—

> A being that is not a mere animal has a language,
> A deaf-mute is a being that is not a mere animal,

Therefore A deaf-mute has a language.

The major premise, as now expressed, is not universal. On the second interpretation the argument will run as follows:

> No mere animal has a language,
> No deaf-mute is a mere animal,

Therefore A deaf-mute has a language.

The argument now violates the rule of the first figure that the minor premise must be affirmative.

> No cat has eight tails,
> One cat has one tail more than no cat,

Therefore One cat has nine tails.

The remarks which were made in connection with the preceding example apply *mutatis mutandis* to the present argument. Moreover, since no term occurs twice in the premises, the argument is not a categorical syllogism, but an elliptical expression of two processes of inference, one of which is an eduction. The full expression of the argument is as follows:

> No cat has eight tails,

Therefore Anything that has one tail more than no cat has nine tails,

> But one cat is a thing that has one tail more than no cat,

Therefore One cat has nine tails.

The middle term, then, is "thing that has one tail more than no cat." The second proposition in the argument as just stated is, of course, not true; besides, it is not a valid eduction from the first proposition, unless "no cat" is interpreted in the sense of "thing that is not a cat."

> All who are chosen are called,
> Few are chosen,
Therefore Few are called.

The sign "few" has the same force as "only a few" (cf. 25). The minor premise is equivalent to the two following propositions: "A few (or some) are chosen." "Most are not chosen." "Most are not chosen," being a negative proposition, cannot be used in the minor premise of the first figure. All that the premises warrant us in concluding is "A few (or some) are called."

131. **Complex** or **Insinuating Question** is a question which assumes as admitted something which has not been admitted.

In Latin the fallacy is called **plures interrogationes,** that is, many questions combined into one. This fallacy consists in so wording a question that a direct answer would place the respondent at an unfair disadvantage. For example: "How long have you been stealing from your employer?" "When were you released from prison?" Frequently the question is so worded as to demand "Yes" or "No" for an answer. Such a question is to be met with a denial of what it assumes (*"Nego suppositum"*). Thus, if a man is asked, "Have you given up cheating in business?" he should answer, "I deny what your question

assumes" or "I deny that I have been cheating in business."

Under this head may be placed the fallacy of **Insinuation** or **Innuendo,** which is generally the expedient of the dishonest or cowardly disputant. It consists in so wording a statement or a question as to create an impression unfavorable to someone, while the question or statement itself, in its literal interpretation, can be acquitted of the charge of libel or of making a direct imputation against the person.

APPENDIX

It must not be supposed that Dr. Keynes' *Formal Logic* has been specially singled out for attack in the following Notes. Far from it. Among English treatises on Logic Dr. Keynes' work is unsurpassed for clearness, conciseness, and thoroughness; and this Handbook is indebted to it in more than one section. The *Formal Logic* is quoted in the following pages because Dr. Keynes expresses his meaning concisely and clearly, thus obviating the need of citing long passages from his work, and making it possible to deal in a few paragraphs with the one or two points on which his doctrine differs from what is laid down in the present volume.

Note on Section 21

THE COPULA

The proposition "The elephant is larger than a horse" is the same as "The elephant is a thing larger than a horse." The predicate is a relative term (cf. 14, 2, 9). In the proposition we assert an objective identity between "elephant" and "thing larger than a horse." There are logicians who claim that the expression "is larger than" is the copula in the foregoing proposition. But this is not the case; for

"larger than" belongs to the *matter* of the proposition, not to the form. "Larger than a horse" is a relative attribute which is predicated of the elephant. That the words "larger than" belong to the predicate, may be plainly seen by submitting the proposition to any of the processes of eduction explained in sections 38, 39, 40, and 41. The Partial Contrapositive and the Inverse of the foregoing proposition are as follows: "No thing (or animal) that is not larger than a horse is an elephant," "Some thing that is not an elephant is a thing not larger than a horse."

Note on Section 29

I.

THE DISTRIBUTION OF THE PREDICATE

Our main reason for not adopting the hypothesis of the distribution of the predicate is that it appears to us to be untenable. The idea which is expressed by the predicate of a proposition is not used distributively in the judgment. We saw in section 8 that an idea is used distributively only when the mind attends to the extension of the idea. In judgment the mind does not attend to the extension of the predicate-idea. Hence, in the predicate of a judgment the idea is not used distributively, but absolutely. This is virtually acknowledged by the advocates of the foregoing doctrine when they reject Sir William Hamilton's

theory of the quantification of the predicate. Hamilton contended that in Logic we should say "All men are some animals," "All men are all rational animals," "No horse is any rational being." Most upholders of the doctrine of the distribution of the predicate say that the predicate is not quantified in thought and therefore its expression should not be quantified. The subject is frequently quantified in thought and hence we may frequently prefix a sign of quantity to it in the proposition. But what is the meaning of the words "quantified in thought"? They cannot mean that the subject-*idea* has a sign of quantity attached to it; for there are no signs of quantity in the mind. The sign of quantity is in the *proposition* to indicate something that is in the thought. The sign of quantity indicates that the extension of the idea is explicitly before the mind; and if we cannot attach a sign of quantity to the predicate of a proposition, this is a proof that the extension of the predicate-idea is not present to the mind in judgment. And if the extension of the predicate-idea is not present to the mind in judgment, then the predicate-idea is not used distributively, but absolutely. Since, then, the predicate-idea is not used distributively in judgment, it is unmeaning to speak of the predicate of a proposition as distributed or undistributed. According to most advocates of the doctrine of the distribution of the predicate, we may put the sign "all" or "no" before the subject when it is distributed, and we may prefix to it the sign "some" when it is undistributed; and yet we are not allowed to do this to the predicate. How, then, can it be maintained that the predicate is sometimes distributed

and sometimes undistributed? The words "distrib-
uted" and "undistributed" surely do not mean one
thing when applied to the subject and something quite
different when applied to the predicate.

The doctrine of the distribution of the predicate is
based upon a confusion of judgment with the mind's
subsequent reflection on it, that is, upon a confusion
of the *meaning* or *import* of a proposition with its
implication (cf. 37). A proposition should contain
only what is explicitly before the mind when the mind
pronounces the judgment expressed by the proposi-
tion. And it is precisely on this ground that the ma-
jority of logicians condemn Hamilton's doctrine of the
quantification of the predicate. When, in opposition
to Hamilton, they say that the predicate is not quanti-
fied in thought, they mean that the extension of the
predicate is not present to the mind. And yet, when
they endeavor to explain the distribution of the predi-
cate, their statements are inconsistent with their argu-
ment against Hamilton. In his *Formal Logic* (4th
edition) Dr. Keynes says on p. 95: "A term is said
to be distributed when reference is made to *all* the
individuals denoted by it; it is said to be undistrib-
uted when they are only referred to *partially*." But
what is meant by the words "reference is made" and
"referred to"? Who or what is it that makes the ref-
erence and does the referring? Is it not the *mind?*
But how can the mind refer to all the individuals de-
noted by a term, unless all the individuals are present
to the mind? And how can the mind refer to the in-
dividuals partially, unless part of the individuals are
before it? The mind cannot make a reference to any-

thing, if it does not know the thing it is referring to;
and it cannot know the thing it is referring to, if the
thing is not present to it. And yet many logicians
condemn Hamilton for saying that the extension of
the predicate is present to the mind, while they them-
selves maintain that the mind refers to the extension
of the predicate. It seems to be impossible to devise
an explanation of the distribution of the predicate
which cannot be used as an argument in favor of
Hamilton's doctrine.

It is only fair to add that, immediately after the
words quoted above, Dr. Keynes says: "That is, [a
term is said to be undistributed] when information
is given with regard to a portion of the class denoted
by the term, but we are left in ignorance with regard
to the remainder of the class." These words may be
so construed as to embrace, not merely the meaning
or import of the proposition, but the implication as
well. If they are intended to include the implication,
the criticism in the preceding paragraph loses some
of its force; but the argument in the paragraphs which
follow will not be affected by this interpretation of Dr.
Keynes' words.

Let it be considered whether the use of the hypothe-
sis of the distribution of the predicate does not resolve
itself into a vicious circle. Let us ask ourselves what
end that hypothesis is intended to serve. The whole
point and purpose of the hypothesis is to enable us
to derive the *implication* of a proposition, to find out
what is *involved* in a proposition. The logician tells
me that the predicate is distributed or undistributed,
and that the knowledge of this will guide me in

inferring the implication of a proposition. But since I am studying Logic, and therefore should understand everything that is laid down in that science, he does not wish me to take his word blindly for the assertion that the predicate is distributed or undistributed; accordingly, he proceeds to prove his assertion. But how does he do it? He does it by appealing to the *implications* of the different propositions, and he shows that in the *implications* the extension of the term is fully or partially referred to. "Therefore," he says, "the term is distributed or undistributed in the predicate of the original proposition, and if you bear this in mind, you will have no difficulty in drawing out the implication of any proposition." "But, my dear sir," I answer, "before you laid down your 'therefore,' I already *knew* the implications of those propositions, and hence I do not need your apparatus. You have deduced the doctrine of the distribution of the predicate from the implications of the propositions, and now you ask me to deduce the implications— *which I already know*—by means of the doctrine of the distribution of the predicate! I do not need your second rule for conversion, because I knew the converse of all the propositions before I understood the rule. Were I to use your hypothesis for the purpose for which it was devised, and *with a real understanding of it,* I should be involved in a vicious circle."

One main purpose of the hypothesis of the distribution of the predicate is to simplify the process of conversion; but it does not simplify that process, if the process must be understood in order to make the hypothesis intelligible.

Let us add a word about the converse of A, E, and I. The logician lays down the doctrine of the distribution of the predicate, and the student accepts it, *after he has seen* what sign of quantity the proposition will have when it is converted. And yet the doctrine was intended to *enable* him to convert. If the logician cannot show from a proposition, as it stands, how its terms are to be understood, but must, in order to this end, refer to the converse, then what he says does not help us to derive the converse. We are accustomed to transfer a designation which a term has in the converse to the term as it stands in the original proposition. This designation is intelligible in the converse; it is not intelligible in the original proposition, unless we recall how the term is designated in the converse.

And now as to the O proposition. When the logician endeavors to show that the predicate is distributed, he refers to the extension of the predicate. In doing this, he is explaining either the import of the proposition, or its implication. If he is explaining the import or meaning of the proposition when he refers to the extension of the predicate, he is bringing us back to Hamilton's doctrine. The import of the proposition is that which is explicitly before the mind in the act of judgment which is expressed by the proposition. If, therefore, the extension of the predicate is found in the import of the proposition, then the extension of the predicate is explicitly before the mind when it makes an act of judgment. But this is exactly the contention of the advocates of Hamilton's theory. On the other hand, if the logician is

explaining the implication of the proposition when he refers to the extension of the predicate, then he should say that the term in the predicate is distributed in the *implication*, not that it is distributed in the original proposition.

It is plain that what has just been said of the O proposition applies also to A, E, and I.

The hypothesis of the distribution of the predicate is based upon an induction. So far as this induction has been carried, the use of the hypothesis leads to a vicious circle. Carry the induction still further, and the same result will be found to ensue.

But there is another objection to the hypothesis of the distribution of the predicate, and that is that it breaks down. The Partial Inverse of "All S is P" is "Some non-S is not P" (cf. 41). Since conversion and obversion are legitimate processes, this partial inverse is a valid conclusion. But according to the foregoing hypothesis, there is a distributed term (P) in the partial inverse which was not distributed in the original proposition, and hence the partial inverse is invalid. Dr. Keynes attempts to get over this difficulty as follows: "It is in the assumption of the existence of the contradictory of the original predicate that an explanation of the apparent anomaly may be found. That assumption may be expressed in the form *Some things are not P.* The conclusion *Some not-S is not P* may accordingly be regarded as based on this premise combined with the explicit premise *All S is P;* and it will be observed that, in the additional premise, *P* is distributed" *(Formal Logic,* p. 140). There are two serious objections to this solution of the difficulty.

In the first place, Dr. Keynes is here using the assumption in a different way from that in which he used it in deriving the obverse and the contrapositive. In deriving the partial inverse he uses the assumption *as a premise,* and that, with the direct purpose of justifying the presence of a distributed term in the partial inverse. This is in conflict with the logical doctrine that eduction (or the so-called immediate inference) is the process of drawing out the implication of a *single* proposition. In the second place, with the aid of the assumption which Dr. Keynes proposes we could, on the hypothesis under discussion, obtain the converse, "All P is S." Thus: "Some things are not animals; All men are animals; Therefore All animals are men." Again, what is to prevent us from justifying in the same way the conclusion of the following argument: "All M is P; No S is M; Therefore No S is P"—"All men are animals; No horses are men; Therefore No horses are animals"? Why can we not maintain that, when we say "All M is P," we may assume that "Some things are not P," and that on the strength of this assumption we may draw the conclusion "No S is P"?

In order to bring out more fully the point we have touched upon in the preceding paragraph, we will suppose the discussion to be carried on by three imaginary characters. Mr. White holds the hypothesis of the distribution of the predicate, but claims the partial inverse of "All S is P" is not valid, because it cannot be squared with that hypothesis. Mr. Brown holds that the partial inverse *is valid,* and he uses this as an objection *against* the *hypothesis.* Mr. Brown

shall speak in behalf of the view set forth in this
Handbook. Mr. Black holds both the hypothesis and
the validity of the partial inverse. Of course we do
not ascribe to Dr. Keynes the mode of defence we
shall put into the mouth of Mr. Black.

Mr. White protests that the partial inverse "Some
non-S is not P" is not valid, because P is distributed
in the partial inverse, whereas it was undistributed in
the original proposition "All S is P."

"But do you not see," says Mr. Black, "that we
must make the assumption 'Some things are not P'
in order to derive the partial inverse? The term P is
distributed in this assumption. The partial inverse
is deduced from *two* propositions, as follows: 'Some
things are not P; All S is P; Therefore Some non-S
is not P.' "

Mr. White replies: "I do not see how you can say
that the partial inverse is the result of a process of
eduction, if you derive it from two propositions."

Mr. Black now turns his attention to Mr. Brown
and remarks: "It is true that the term 'animal' is
distributed in your premises, and this warrants you
in distributing that term in your conclusion; but you
have no right to violate the other rules which must be ob-
served to insure validity. You have drawn the *affirmative*
conclusion 'All animals are men' from the premises
'Some things are not animals, All men are animals,'
one of which is *negative*."

"Well, then," answers Mr. Brown, "let me construct
another argument, the conclusion of which follows
rigorously according to the hypothesis of the distribu-
tion of the predicate and your interpretation of that

hypothesis, and which is not even apparently open to
your criticism.   The argument is as follows:

> Some things are not animals,
> All men are animals,
> . Therefore some men are not animals.

You will observe that not one of the general rules
of the syllogism is violated in this argument.   It is
true that the argument offends against the rule which
is sometimes given, viz. 'The conclusion must not
contain the middle term;' but this rule is also violated
in the argument you employed above when you de-
rived the partial inverse, viz.:

> Some things are not P,
> All S is P,
> Therefore some non-S is not P.

Moreover, there are four terms in your argument, viz.
'S,' 'P,' 'thing,' and 'non-S.'

"But your criticism," Mr. Brown continues, "has
only an apparent, not a real, force against my conten-
tion.   Let me ask what it was that you required in your
premises in order to draw the conclusion 'Some non-S
is not P.'   Did you require a negative premise?   Not
at all.   Your conclusion is not made negative by the
premise 'Some things are not P.'   It is made negative
by the process of obversion which is performed on the
inverse 'Some non-S is non-P.'   The effect of obversion
on an affirmative proposition is always to make it neg-
ative; and if it does not make the proposition negative,
the process is invalid.   Hence, you were not looking
for a negative premise in order to justify your con-
clusion.   The assumption 'All P is S' would have

served your purpose just as well as the assumption
you resorted to, if you could have given any plausible
reason for assuming 'All P is S.'   As a matter of fact,
I do not see why you do not employ the assumption
'All P is P,' which is absolutely self-evident, and the
use of which would rid your argument of the fallacy
of four terms.

"Since, therefore, your conclusion 'Some non-S is
not P' is not made negative by the premise 'Some
things are not P,' you have no right to claim that
that premise should negative the conclusion 'All ani-
mals are men.'   If the force of that premise is to
negative the conclusion, then your conclusion should
be affirmative; for the proposition in your conclusion
was made negative by the process of obversion; and
if that negative premise is to have a negative effect
upon this negative proposition, then the negative con-
clusion should be changed into an affirmative.

"The one and only thing that you were looking for,
in order to justify the partial inverse to Mr. White,
was that the term P should be *distributed* in your
premises; and it was altogether immaterial to the
purpose of your proof whether P was distributed in
an affirmative or in a negative premise.   Similarly,
the only thing that is necessary for my proof of the
validity of the conclusion 'All animals are men,' ac-
cording to the hypothesis of the distribution of the
predicate and your use of that hypothesis, is that
'animal' be distributed in my premises; and it is irrele-
vant to complain that one of my premises is negative.
However, if it still be insisted that my conclusion
must be negative, I will make it negative, and write

it thus: 'Some men are not animals.' This conclusion serves my purpose as perfectly as the one I originally inferred.

"The obverted converse of 'Some S is P' is 'Some P is not non-S':—where did you get this 'non-S'? and how does it come to be 'distributed'? The only assumption you had to aid you in deriving this obverted converse was 'Some things are non-S;' and in this assumption 'non-S' is *undistributed.* Perhaps you will say that you did not get the 'distributed' 'non-S' from 'Some things are non-S,' but that you got it by obverting the assumption 'Some things are S.' But that is only pushing the difficulty one step farther back; for if you say that you got the 'distributed' 'non-S' by obverting 'Some things are S,' I have three questions to put to you: First, how did you get the 'non-S,' since it is not contained in 'Some things are S'? Secondly, how does 'non-S' come to be 'distributed' in 'Some things are not non-S,' when it was not 'distributed' in 'Some things are S'? Thirdly, if you can by the mere process of obversion change the quality of a proposition, and obtain in the obverse a 'distributed' term which was neither 'distributed' nor contained in the proposition you obverted, why are you not content to say that the process of obversion is sufficient to account for the 'distributed' P in the partial inverse 'Some non-S is not P'? Why do you insist upon bringing in the assumption 'Some things are not P'— I do not say, to enable you to assert the partial inverse absolutely, for that is not the point of our discussion, and it applies also to the full inverse, but—to explain why the term P is *distributed* in the partial

inverse? If the process of obversion explains the
thing in one case, why does it not explain it in the
other?

"Dr. Keynes says: 'We assume that *S, not-S, P,
not-P,* all represent existing classes' *(Formal Logic,*
p. 139). That is, we assume the following proposi-
tions: 'Some things are S,' 'Some things are non-S,'
'Some things are P,' 'Some things are non-P.' The
converse of 'Some S is P' is 'Some P is S,' and the
partial contrapositive of 'Some S is not P' is 'Some
non-P is S.' To which of the foregoing assumptions
did we have recourse, in order to derive this converse
and this partial contrapositive? No matter which one
we selected, we had two particular premises; and from
two particular premises no conclusion can be drawn."

"But," answers Mr. Black, "we do not *need* any of
those assumptions to prove the validity of the con-
verse 'Some P is S;' and, though we have to assume
it to be true that 'Some things are non-P,' in order
to justify the partial contrapositive 'Some non-P is S,'
still *we do not have to use this assumption as a prem-
ise.* And besides, you are forgetting that we are
dealing here with *eduction,* which is the process of
drawing out the implication of a *single* proposition.
Let me put my answer in another way, as follows: I
know that the converse is valid, and no one who holds
the hypothesis of the distribution of the predicate
questions its validity; therefore, I will not use any
of those assumptions as a premise. I *know* that the
partial contrapositive is valid; but I do not need to use
any of those assumptions *as a premise,* in order to derive
it. In fact, if I did use one of them, everyone would

say that the partial contrapositive was invalid. I *know*
that the partial inverse 'Some non-S is not P' is valid; but
it makes the hypothesis of the distribution of the
predicate look queer. But since that hypothesis can-
not possibly be false, and since I cannot hold it and
at the same time maintain the validity of the partial
inverse, without using one of those assumptions as a
premise, therefore, I will use one of them as a premise.
As to the statement which I made above, viz. that
eduction is the process of drawing out the implication
of a *single* proposition,—well,—that has no force here;
because, unless I use one of the assumptions as a
premise, I cannot defend the hypothesis of the distri-
bution of the predicate."

Mr. Brown replies: "Our discussion really amounts
to this: I bring an objection against the hypothesis
of the distribution of the predicate; and, in order to
answer it, you say we must reject the ordinary doc-
trine that eduction is the process of drawing out the
implication of a single proposition; and the result of '
your defence of the hypothesis is that we are justified
in deducing 'Some men are not animals' from the
proposition 'All men are animals.'"

We should on every account be sorry to have mis-
represented the position of those who try to reconcile
the partial inverse of "All S is P" with the hypothesis
of the distribution of the predicate. If in the fore-
going discussion we have been demolishing a man of
straw, we shall be only too glad to acknowledge it.
But we think the force of the following dilemma
will have to be admitted: Either "Some non-S is not
P" is an implication of the proposition "All S is P,"

or it is not. If it is, then the hypothesis of the distri-
bution of the predicate breaks down. If it is not, then
the advocates of that hypothesis are wrong in calling
it an implication. But of course it would be simply beg-
ging the question in favor of the hypothesis, if anyone
refused to call the partial inverse an implication of "All
S is P" on the ground that it could not be reconciled
with the hypothesis. We might be willing to allow that,
in order to assert the partial inverse absolutely, it may
be necessary to use one of the assumptions as a prem-
ise; but then we shall maintain that this is necessary
in the case of the full inverse; and when once it has been
done for the full inverse, there is no need of doing it
again for the partial inverse. Moreover, we shall be
prepared to maintain that the use of the assumption as
a premise will be of no assistance in accounting for the
"distributed" term P in the partial inverse.

II.

CLASS MODE OF READING PROPOSITIONS: QUANTIFICATION OF THE PREDICATE

Perhaps some remark should be made here upon the
class mode of interpreting the categorical proposition.
When we speak of a term as distributed or undistributed,
it is plain that we have in mind the *distributive,* not
the *collective,* use of the term (cf. 18); that is, we are
thinking of the objects denoted by the term, not as a
collection or group; we are thinking of them one by
one. But when we speak of the subject and predi-

cate of a proposition as classes, we are not thinking of the objects one by one; we are thinking of them as collections; and this means that we are using the subject-term and the predicate-term *collectively*. Consequently, we have no right in this case to speak of either term as distributed or undistributed.

It is notorious that the sign "all" is ambiguous. We cannot tell merely from the sign "all" whether the subject-term is used distributively or collectively. For this reason, logicians warn us that the sign "every" should be employed whenever there is a chance that the distributive use of the subject-term may be mistaken for the collective. Whenever the subject-term is used distributively, the sign "every" may be substituted for "all" with absolutely no change of meaning or force. If, therefore, we would avoid fallacy in discussing the interpretation of categorical propositions, we should always replace the sign "all" by "every." The application of this test will lead to a summary settlement of one or two controverted points in Logic.

The foregoing consideration disposes of Hamilton's contention that the proposition "All men are rational animals" should read "All men are all rational animals." If we substitute "every" for "all" in Hamilton's proposition, we have "Every man is every rational animal."

What has just been said of the sign "all" is true *mutatis mutandis* of the sign "some." In Logic this word has the same force as the expression "a certain (or uncertain) number of," and, when attached to the subject, it may have either a distributive or a collective

force. It has a collective force in the proposition "Some carpenters are building the barn." When the use is collective, the force of "some" is "a certain number of . . . together." When the use is distributive, "some" has the force of "a certain number of . . . individually" or "everyone of a certain number of." The converse of the proposition "Every man is an animal" or "All men are animals" is "Some animals are men," and this is the same as "Everyone of a certain number of animals is a man." Let us now take Hamilton's proposition, "All men are some animals." If we interpret the terms of this proposition distributively, and not collectively, the proposition will read, "Every man is everyone of a certain number of animals."

Unless judgment and classification are one and the same thing, the "class-inclusion" or "class-exclusion" interpretation of judgment and the device of illustrating judgments by means of circles proceed on a false hypothesis. Judgment and classification are obviously not one and the same thing. Judgment is a single act of the mind; classification is a process and involves inference. In order to think of an object as a member of a class, we must first judge that it has the attribute which will warrant us in classifying it with the other members. Classification may issue in a judgment, as a process of inference frequently does; but even then all the elements which were present in the process of classification do not enter into the judgment.

The remark which was made on page 205 concerning the O proposition should be applied to diagrammatic illustrations of judgments and categorical propositions.

If these diagrams are intended to illustrate the propositions themselves, that is, the meaning or import of the propositions, then the diagrams only serve to recommend the theory of Hamilton. If they are intended to illustrate the implication of the propositions, the diagrams are, to say the least, very misleading, and they do but complicate what they were designed to simplify.

Note on Section 45

IMMEDIATE INFERENCES

The processes of Eduction which were explained in Chapter V are commonly called Immediate Inferences. If by "inference" is meant the *process* of inference, we have no more right to apply the name "immediate inference" to these processes than we have to apply it to the Mixed Hypothetical Syllogism. The process of inference issues in the assertion of the consequent (cf. 54). If Conversion is to result in the assertion of the Converse, the full expression of the process requires that the act of inference be stated as the major premise; thus: "If no men are immortal, then no immortal beings are men; But no men are immortal; Therefore no immortal beings are men." The major premise is necessary to the complete expression of the process, because the assertion expressed by this premise is what would be denied by a person who should deny the validity of the converse or conclusion. If we were to say "Some Frenchmen are not scientists; Therefore some scientists are not Frenchmen;" the logician would not deny the proposition which occupies the position of conclusion; for he

knows that it is true. He would deny the sequence, that is, he would deny the suppressed major premise, namely, "If some Frenchmen are not scientists, then some scientists are not Frenchmen." "No men are immortal; Therefore no immortal beings are men:"—this is no more a full account of the mental process than, in another subject-matter, is the following: "The defendant is innocent; Therefore the judge should acquit him."

Note on Section 55

THE DEFINITION OF THE SYLLOGISM

The definition of the Syllogism in section 55 does not apply to the ordinary mode of expressing the process by which the conclusion is reached in the Categorical Syllogism or in the *Modus Tollens* of the Mixed Hypothetical Syllogism; but the definition does apply in all strictness to the *full expression* of the process underlying the Categorical Syllogism, as will be made clear in the latter part of this Note. The definition also applies strictly to the full expression of the process which immediately terminates in the conclusion of the *Modus Tollens*. Let us take as an example the following argument: "If it has rained, the grass is wet; But the grass is not wet; Therefore it has not rained." In the process by which the conclusion is reached in this argument we have apparently the following order: the act of inference, the denial of the consequent, and the denial of the antecedent; whereas the definition of the Syllogism given in section 55 requires the order to be as follows: the act of inference, the

assertion of the antecedent, and the assertion of the
consequent. The explanation of this seeming incon-
sistency is that we have in the *Modus Tollens* an ellip-
tical expression of *two* processes of inference. In
order to draw the conclusion, "Therefore it has not
rained," we must first recognize that the falsity of the
antecedent is logically dependent upon the falsity of
the consequent; and this recognition gives us an addi-
tional act of inference, viz. "If the grass is not wet,
it has not rained." Hence, the full expression of the
original argument is as follows:

> If it has rained, the grass is wet,
> Therefore, if the grass is not wet, it has
>   not rained,
> But the grass is not wet,
> Therefore it has not rained.

If a person did not recognize that the second act of
inference is a valid conclusion from the first, he might
assert on the authority of others that we are war-
ranted in drawing the conclusion, "Therefore it has
not rained," but he would not understand *why* we are
warranted in doing so. It will be observed that the
antecedent of the second act of inference is asserted
in the minor premise, and that the consequent is as-
serted in the conclusion. The true major premise of
the conclusion, "Therefore it has not rained," is as
follows: "If the grass is not wet, it has not rained."
Offer a person the following enthymeme: "The grass
is not wet; Therefore it has not rained;" and ask him
to supply the major premise. It will be found that he
will say "If the grass is not wet, it has not rained."

He will not think of putting down "If it has rained, the grass is wet" as the major premise of the enthymeme, any more than he will think of writing "If the grass is not wet, it has not rained" as the major premise of the following enthymeme: "It has rained; Therefore the grass is wet." Hence, the argument of which we have been speaking is in reality the expression of two processes of inference. However, for the sake of simplicity, it is usually called merely a syllogism. We have followed this convenient terminology.

As regards the Categorical Syllogism, the following points are to be noticed: All the rules and the entire explanation of the Categorical Syllogism are concerned with the *act* of inference, and not at all with the *process*. The rules are not rules for reaching a *conclusion,* but only for determining a *sequence.* The rules tell us when the consequent is involved in the antecedent, but they do not tell us what we have to do to *prove* the consequent and come to a *conclusion.* Now, every process of inference must contain an act of inference; and if the act of inference is not expressed, we have only an elliptical expression of the process of inference. In the ordinary method of writing the Categorical Syllogism the act of inference is not expressed. If the act of inference is expressed, it will be seen that our definition of the Syllogism fits the Categorical Syllogism perfectly. For example:

> If all men are mortal and all kings are men,
>> then all kings are mortal,
> But all men are mortal and all kings are men,
> Therefore all kings are mortal.

Nor can it be maintained that the particle "therefore" expresses the act of inference. It indicates in a general way that there was a sequence, but it cannot in any way be regarded as the *formal expression* of the sequence. In order to be the formal expression of the sequence, it would have to be exactly equivalent to "If . . . then;" and it certainly has not the force of these particles. "Therefore" does a great deal more than indicate a sequence; its peculiar force consists in this: it sets forth formally that the conclusion has been *proved*. "Therefore" is formally the mark of a *conclusion*, not of a sequence. Besides, the particle "therefore" appears in the conclusion of the Mixed Hypothetical Syllogism; and no one would consider this type of syllogism a complete expression of a process of inference, if the hypothetical premise were omitted. And yet the hypothetical premise would be unnecessary, if the particle "therefore" were the formal expression of the act of inference. But the particle "therefore" has not a different force in the Categorical Syllogism from what it has in the Mixed Hypothetical.

Note on Section 74

THE FOURTH FIGURE

Dr. Keynes makes the following observation concerning the fourth figure: "It [the fourth figure] yields conclusions which are not directly obtainable from the same premises in any other figure. It is not actually in frequent use, but reasonings may sometimes

not unnaturally fall into it; for example, None of the Apostles were Greeks, Some Greeks are worthy of all honor, therefore, Some worthy of all honor are not Apostles" *(Formal Logic*, p. 329).

The only reason why Dr. Keynes' contention may seem to have some plausibility is because the major premise in his example would have an unnatural appearance if it were converted. The converse of this premise would be, "No Greeks were any of the Apostles." This unnaturalness is due to the fact that the major premise is an *enumerative* universal, not an unconditional one. The conclusion which Dr. Keynes infers would be warranted only on the supposition that the major premise was not an enumerative universal, —at least in form,—but that it was, "No Apostles were Greeks." The conclusion from Dr. Keynes' premises ought to be, "Some worthy of all honor are not any of the Apostles."

If we are to account the fourth figure a natural form of argument and employ in it the ordinary fourfold scheme of categorical propositions, we ought to be able to produce concrete examples of natural syllogisms in the fourth figure without departing from the fourfold scheme. Supposing a man wished to prove the proposition, "Some worthy of all honor are not Apostles," his argument would naturally take some such form as the following: "No Greeks were Apostles, Some Greeks are worthy of all honor, therefore, Some worthy of all honor are not Apostles." It would be unnatural for him to start with the proposition, "No Apostles were Greeks," and retain the same minor premise.

Note on Section 80

RELATIVE TERMS IN THE CATEGORICAL SYLLOGISM

Dr. Keynes remarks as follows on the explanation of the first example in section 80: "We may treat this as a mere evasion, or as a *petitio principii.* The principle of the argument *à fortiori* is really assumed in passing from ['The horse is (a thing) larger than a dog'] to ['Every thing larger than a horse is (a thing) larger than a dog']" *(Formal Logic,* p. 385). Doubtless it is assumed, just as the Three Fundamental Laws of Thought are assumed. The *à fortiori* principle cannot be proved. If it is not accepted as self-evident, it is useless to discuss it. But we do not attempt to evade the *à fortiori* principle when we express the foregoing eduction, viz. "Every thing larger than a horse is (a thing) larger than a dog." This eduction is expressed in order to bring out in words what was in the thought; and this is what is always done when the missing proposition of an enthymeme is supplied. Unless we indicate in language the term or basis of comparison—viz. "thing larger than a horse"—which enabled the mind to combine the subject and predicate of the conclusion, and unless we insert this term in two premises, we have not given full expression to the mental process.

In order to provide for arguments like the one we have been discussing, it has been proposed to supplement the *dictum* of the first figure with the following

axiom: "Whatever is greater than a second thing which is greater than a third thing is itself greater than that third thing." But since there is an endless variety of relations, we should be obliged to provide an endless number of axioms. Thus, we should need a different axiom for the following argument:

> The elephant is larger than a horse,
> The dog is smaller than a horse,
> Therefore The dog is not as large as an elephant.

The major premise, which is not expressed, and which was derived by eduction from "The elephant is larger than a horse," is as follows: "Nothing smaller than a horse is as large as an elephant." But the *dictum* does not need to be supplemented. An attribute or an idea *is* an attribute or an idea, whether it be absolute or relative. If it be desired to make special provision for relative attributes, all we have to do is to substitute "relative attribute" and "relative idea" for "attribute" and "idea" respectively in any of the five *dicta*. For example: "Any relative attribute [P] which is affirmed or denied of the formal object [M] of a universal relative idea may be affirmed or denied respectively of anything [S] which possesses that formal object [M]." Cf. *Note on Section 21*. The following is an example of a syllogism in the second figure, in which the major and middle terms are relative:

> Nothing that is as large as an elephant is as small as a horse,
> The mule is as small as a horse,
> Therefore The mule is not as large as an elephant.

Note on Section 93

THE PURE HYPOTHETICAL SYLLOGISM

In section 93 we said that the antecedent and consequent of the conclusion in the Pure Hypothetical Syllogism correspond in general to the minor and major terms respectively of the Categorical Syllogism, and the potential judgment which appears only in the premises, to the middle term. It will be observed, however, that in the second figure of the Pure Hypothetical Syllogism there are apparently five potential judgments. For example—

<div align="center">

If A is B, C is D,

If E is F, C is not D,

Therefore If E is F, A is not B.

</div>

The potential judgments in the foregoing syllogism are the following: A is B, C is D, E is F, C is not D, A is not B.

It must be borne in mind that the matter of a hypothetical proposition is not the same as that of a categorical. The matter of a hypothetical proposition are the formal objects of potential judgments, each of which may be pronounced true or may be pronounced not true. To negative an affirmative judgment is the same as to assert its contradictory, that is, to make a negative judgment. To negative a negative judgment is the same as to assert its contradictory, that is, to make an affirmative judgment.

If we let X stand for the judgment "Peter is ill," then the judgment "X is true" is the same as "Peter is ill;" and the judgment "X is not true" is the same

as "Peter is not ill." Again, if we let Y stand for the judgment "James is not a lawyer," then the judgment "Y is true" is the same as "James is not a lawyer;" and the judgment "Y is not true" is the same as "James is a lawyer."

Let us now represent "A is B," "C is D," and "E is F" in the foregoing syllogism by the letters C, M, and A respectively. The syllogism will then read as follows:

<div style="text-align:center">

If C is true, M is true,

If A is true, M is not true,

Therefore If A is true, C is not true.

</div>

It will thus be seen that even in the second figure of the Pure Hypothetical Syllogism we are dealing in reality with three elements, and that one of these elements affords a basis of comparison for the others.

The following are the *dicta* of the three figures of the Pure Hypothetical Syllogism:

Dictum of the First Figure: "Any potential judgment implied (involved) in another which is implied in a third is itself implied in the third."

Dictum of the Second Figure: "When one potential judgment implies another whose contradictory is implied in a third, the contradictory of the first is implied in the third."

Dictum of the Third Figure: "Any potential judgment implied in another which implies a third is itself sometimes implied in the third (*or* may itself be implied in the third)."

REFERENCES

The following references will be useful to those who wish to undertake a deeper study of Logic or who are looking for fresh illustrations or practical exercises:

Keynes, *Formal Logic* (4th edit., London, 1906).
Joyce, *Principles of Logic* (London, 1908).
Coffey, *Science of Logic* (London, 1912).
Joseph, *Introduction to Logic* (2d edit., Oxford, 1916).
Whately, *Elements of Logic* (8th edit., London).
Jevons, *Elementary Lessons in Logic* (5th edit., London, 1875).
Jevons, *Studies in Deductive Logic* (London, 1884).
Hibben, *Logic Deductive and Inductive* (New York, 1905).
Fowler, *Elements of Deductive Logic* (Oxford, 1895).
Creighton, *Introductory Logic* (3d edit., New York, 1909).
Turner, *Lessons in Logic* (Washington, 1911).
Clarke, *Logic* (London).

This Handbook is indebted for a number of its examples to the works of the following authors: Keynes, Joyce, Coffey, Joseph, Whately, Jevons, Creighton, and Clarke.

INDEX

229

MAY 8   1919